Article

Ultrasonic Transceiver with a Regular/Periodic 1-3 Piezocomposite based on the SAW Resonance Mode on Damping Backing

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Abstract — 1-3 piezocomposite is widely used as a low acoustical impedance, high anisotropy and transceiver efficiency active element, particularly for ultrasonic imaging, fluid flow measurements, etc.

A novel effective vibrational mode was found in the conventional transducer with array of orthogonal (square) regular piezo-rods in 1-3 piezocomposite, containing the damping backing and front matching layers. The operational resonance in the structure is the Surface Acoustic Wave (*SAW*) on the backing boundary excited by the adjacent piezo-rods, with a frequency typically near 3 times lower the fundamental half-lambda piezocomposite resonance, and which is located near the minimum of the transducer resistance (real part of the complex impedance).

Pulse-echo sensitivity and transmitting sound-pressure-level (*SPL*) showed the signal strength roughly near -83dB_re_1V@1μbar and 128dB_re_20μPa@4"@150kHz@400V_{0-p} in air, comparable to the conventional similar air transducers at the frequency range 0.1...0.7 MHz, where at these frequencies the lateral and longitudinal piezoelement dimensions are typically close to each other causing interference with unwanted coupling modes.

The backing *SAW* resonance effect in the transducer performance is inherent just to the regular periodic 1-3 piezocomposite structure, and does occur neither with randomly located/oriented piezo-rods, nor in the homogeneous piezo-plate at least with the same lateral cross-section as the connected to it backing.

The investigated phenomena can improve the transceiver sensitivity, bandwidth, providing lower drive voltage and smaller and lighter weight acoustic transducers.

Based on the piezocomposites with thickness' 1...1.5 mm (rod resonance near 2...3 MHz), pillar width 0.2...0.8 mm, kerf width 0.1...0.4 mm, resulting in a volume fraction of piezoceramic (PZT-5A) 40-60%, the transducers with an operating frequency from ~140 kHz to ~500 kHz were designed and fabricated with a conventional backing of a mixture of high-density tungsten powder and epoxy, and a matching layer of a mixture of low-density glass bubbles and epoxy.

Keywords: piezoelectric materials; piezoelement; 1-3 piezocomposite; resonance and antiresonance frequencies; quality *Q*-factor, surface acoustic wave (*SAW*), interdigital transducer (*IDT*), mass load, *SAW* resonator

1. Introduction

The 1-3 piezocomposite consists of an array of parallel piezoelectric rods embedded in a polymer matrix with electrode layers on the top and bottom surfaces, and incorporates epoxy('ies) for bondlines, backing and matching layers — in order to obtain improved mechanical and ultrasonic performance compared to a baseline homogeneous plate of material, for example. It provides broader bandwidth, shorter pulse, lower *Q*-factor, clear spectra with suppressed planar unwanted modes. In conventional piezocomposite applications, the height of the pillars normally is about one half wavelength at the operating frequency if the backing and matching materials are both lower in acoustic impedance. The piezoelectric layer in the structure is an active resonating transducing element, which converts electrical energy to acoustical energy and vice versa. The most common piezoelectric materials used in ultrasound transducers are piezoceramics such as lead zirconate titanate (Pb(Zr,Ti)O₃, or PZT), and based on it piezocomposites, which possess relatively strong piezoelectric effect [1-3]. In the traditional operation, a voltage pulse is applied between the front and back surfaces of the ceramic rods of particularly a 1-3 piezocomposite to excite a mechanical expansion thickness resonance (rod length) of the piezoelectric plate.

The transducer generally works as a transceiver, generating and receiving the pressure pulses typically at the transducer half-lambda resonant frequency F_r evaluated [2,4] as:

$$F_r = c/2L \tag{1}$$

where L is the thickness of piezoelectric layer and c is the speed of sound (SOS) for the thickness mode in piezoelectric material. For a thin homogeneous plate the latter parameter $c = \sqrt{C_{33}^E/\rho}$, and for a 1-3 piezocomposite $c = \sqrt{1/\rho S_{33}^E}$, with C_{33}^E and $1/S_{33}^E$ taken as elastic stiffness and Young modulus under short-circuit condition, ρ is the material density. For a given material, a 2 MHz array needs approximately 1.0 mm thick ceramic layer.

Conventionally, the planar (redial) modes typically are used for a low frequency range below near 100 kHz, while for a high frequency range above near 700 kHz a thickness vibrational mode of a monolithic, or piezocomposite, plate is exploited. For intermedium frequencies a similar piezocomposite structure still is a good solution. Also a radial mode of a relatively thin conventional piezoelement sometimes is used as well, however it's not so efficient because of a Poison coefficient effect with near 1/3 decrease of displacement in the longitudinal direction [5]. In the present research, a new operational vibrational mode of a SAW type in 1-3 regular piezocomposite, other than piezoelectric resonance, was proposed and investigated.

Typically a quarter-lambda wavelength thick matching layer is introduced to provide the acoustic matching between the ceramic and fluid. The impedance of air is as low as near 100 Rayls, so that a

light couplant is used to reduce this impedance mismatch. For a transducer with a single matching layer, its optimal acoustic impedance Z_m value is a geometrical mean [4] $Z_m = \sqrt{Z_0 Z_R}$, where Z_0 and Z_R is the acoustical impedance of piezoelectric material and fluid, respectively. The effect of perfectly matching layer is the maximum efficient energy transmission at the center frequency, providing relatively wide transducer bandwidth. Implementing typically light glass bubbles into epoxies is common practice to form matching material, when the acoustic impedance can be adjusted by the percentage of loaded powder to meet the design requirements of transducers [6].

The air attenuation is frequency-dependent significantly increasing with frequency, when particularly the air acoustical attenuation at 0.5 MHz is near 4 times less than that at 1 MHz, so that typically there is a trade-off between transceiver sensitivity and resolution.

The energy generated in the transducer can radiate in both the forward direction and reverse direction. The purpose of the matching layer is to encourage energy to be propagated in the forward direction with low loss and reflections. On the contrary, the backing layer is designed to maximum attenuate the signal emanating from the back surface of the piezoelectric layer, as well as reduce ringing. Ringing is caused by residual energy resonating within the piezoelectric layer, and so it can be reduced by allowing that energy to be coupled out and attenuated in the backing. If the attenuation of the backing material is sufficiently large, no reflections from the back surface of backing layer can be found [2].

To minimize the ringing, the acoustic impedance of backing layer can be matched to piezoelectric material. But as a result, half of the energy will be transferred into the backing layer and then lost. A very short pulse could be obtained, but with a relatively low amplitude. Thus, compromise is always taken between sensitivity and bandwidth. Therefore, the acoustic impedance of backing layer is usually slightly lower than the one of piezoelectric layer in order to improve sensitivity at the cost of slightly increased pulse length or ringing – for some particular applications a predetermined waveform, with a definite number of the sine peaks, should be provided. The epoxies loaded with fine dense powders and heavy chips such as tungsten are commonly used as backing materials [2]. The purpose of loading powder into epoxies is to change the acoustic impedance similarly to the technique used for matching layer design, but with opposite effect of increased density, creating multiple centers of energy dissipation.

The general benefits of the 1-3 piezocomposite structure are higher thickness mode piezoactivity (rods k_{33} -type vibration), lower frequency transverse (planar) unwanted modes, lower acoustical impedance and transducer Q-factor. Several unwanted higher frequency resonances caused by the

regular piezoceramic-epoxy structure in piezocomposites, are found and described in [7-11]. To damp the unwanted resonances, an irregular piezocomposite structure is recommended as in [12-14].

For the acoustical performance characterization of the conventional layered transceiver, the transmission line (1-D approach) theory is typically considered. It particularly states that in a two-layer structure, like piezoelement and backing, there is a smooth *single*-mode transition between fundamental $\lambda/2$ to $\lambda/4$ modes of the piezoelement when the backing acoustical impedance goes from a relatively low value to high, without possibility for double modes simultaneous excitation. So, when the piezoelement and backing acoustical impedances are near equal, which is the perfect matching condition, the total structure behave itself as a single body, without revealing simultaneously the $\lambda/2$ and $\lambda/4$ fundamental resonances of the piezoelement. Additionally, as the transmission line approach is one-dimensional, it directly suggests and relates to the averaged (effective), or homogeneous piezoelement and backing bodies of the same transverse area.

In case of a 1-3 piezocomposite element (Figure 1a), it is structured with typically periodic piezoelectric rods imbedded into softer and lighter epoxy. In the conventional approach, the effective acoustical impedance of such a structure is proportional to the structural effective (averaged) density, following to the *kerf-to-pitch* piezocomposite parameter. Typically, the effective acoustical impedance of a piezocomposite is near 2 times lower than that in piezoceramics.

To the contrary of the conventional approach for the total piezocomposite structure as a whole, a single piezoelectric rod attached to a transversely wider backing was considered here to describe the rods individual and collective behavior on the backing.

2. Boundary SAW Resonance Performance of the Piezoelectric 1-3 Composite Bonded to Backing

2.1. 1-3 Piezocomposites with Regular Structure for the experiments

In the present research, a novel acoustically effective vibrational mode in the 1-3 piezoelectric transducer was found, described and investigated. The resonance mode vibration is based on the SAW on the backing bonding boundary and has its frequency significantly lower the piezocomposite fundamental ($\lambda/2$ mode) thickness resonance. In this case the typical operation transducer frequency (piezocomposite with backing and matching) is near 3 times lower than the conventional piezocomposite thickness resonance (Figure 1b and Figure 1c).

As was concluded, there are two critical factors supporting the transducer *SAW* operation with that new condition: necessary regular periodic transverse structure in the 1-3 piezocomposite, and a solid

layer of bonding material (hard epoxy) between 1-3 piezocomposite and backing. The transducer has a conventional damping backing with near 10...20 MRayls acoustical impedance; conventional low impedance matching layer (glass bubbles with epoxy), with its quarter-lambda resonance at the *SAW* resonance frequency. All that provides the transceiver operation close to the minimum transducer resistance (real part of the impedance), comparable to the piezocomposite impedance at its thickness resonance.

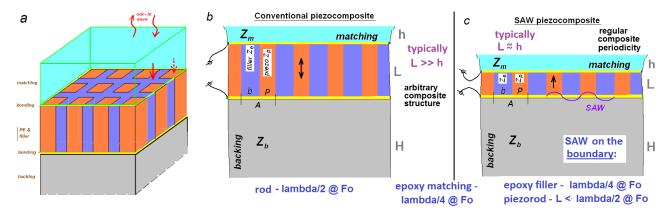


Figure 1. Conventional 1-3 epoxy-filled PZT piezocomposite, with low-Q, high anisotropy, that is used to provide the acoustical performance with low acoustical impedance, high damping and radial mode suppression: a – general view; b –structure for the thickness resonance mode; c –structure for the SAW resonance mode of the same operation frequency.

Practical realization of the frequency range with near 100...500 kHz operation frequency in a conventional transducer design is complicated due to the necessity to use a piezoelement with close lateral and transverse dimensions, causing complex transducer spectra and relatively lower acoustical efficiency.

In the present experiments, the 1-3 piezocomposites have near 65% volume of PZT-5A type (CeramTec GmbH, Germany) [3] piezoceramic, with effective acoustical impedance near 17 MRayl. They have typically 12 x 7 mm planar dimensions, with near 2.5 MHz piezoelement thickness resonance.

Structural parameters of the piezocomposite samples used in the experiments are shown in Table I, all values further are presented in SI.

Table I. Structural array piezocomposite parameters, used in experiments.

#	L, mm	<i>p</i> , μ	<i>b</i> , μ	v+b, mm	L/p	L/p+b	$p^2/(p+b)^2$	F, kHz
1	0.76	120	80	0.20	6.3	3.8	0.36	~500 kHz
2	0.63	180	80	0.26	3.5	2.4	0.48	~500 kHz
3	0.88	540	160	0.70	1.6	1.3	0.60	~500 kHz
D1 / D2	2.7/5.0	1000	400	1.40	2.7/5.0	23	0.51	~150 kHz

Reference info: typically L/p > 1; piezomodule d_{33} 440...520 pC/N; capacitance 1900...2800 pF typ.; samples numbering is arranged in ascending order with the pillar (pitch) values; for D, two different elements.

Relative ceramic volume (effective density) in piezocomposite is determined by the parameter $p^2/(p+b)^2$, and the effective density can be expresses as

$$eff \frac{\rho_{composite}}{\rho_{ceramic}} = \frac{\rho_{ep}}{\rho_{PE}} + \frac{p^2}{(p+b)^2} \left(1 - \frac{\rho_{ep}}{\rho_{PE}}\right). \tag{2}$$

The backing with a height H of several wave-lengths at the operation (SAW) frequency was a typical dense mixture of tungsten powder and chips with epoxy – it provides effective backing with acoustical impedance near 15 MRayls and Q-factor near 5 [15]. The quarter-lambda thick matching layer was a mixture of glass-bubbles with epoxy, providing relatively low acoustical impedance near 1 MRayl. Basic parameters of the materials used in the experiments are shown in Table II.

For the acoustical tests, the distance ~6.4 cm was used in pitch-catch testing, and a reflective SS block with a distance 2.5" was used as a target in pulse-echo tests, both in the transducer "far field" area.

Table II. Transcerver infactions Data											
	E, GPa	ρ , kg/m ³	c, m/s	Zac, MRayl	σ	1/Q					
Piezoceramic	122	7800	3960 (rod)	30	0.35	1/100					
Epoxy in composite	8.1	1200	2600 (rod)	3	~0.40	1/10					
Matching (glass bubbles&epoxy)	2.4	390	2500 (plate)	0.9	~0.40	1/5					
Backing (W powder&epoxy)	22	13000	1300 (600 SAW)	17	~0.35	1/6					
Air	0.00015	1.25	350	0.00045	_	_					

Table II. Transceiver Materials Data

where E – Young's/bulk module; ρ – density; c – SOS; Z_{ac} – acoustical impedance; σ – Poisson coefficient; 1/Q – attenuation coefficient, with the quality factor Q.

An excitation spike of 300V amplitude was applied to the structure, so that with an output signal typically 100 mV, it provides near -70 dB pulse-echo sensitivity. In the basic characterization, together with the fundamental thickness resonance, there are noticed typical higher frequency spurious resonances, related to a Lamb wave in just a piezocomposite plate, with rods filled with epoxy.

2.2. Effect of SAW on the Backing Boundary - Prediction

A regular piezo-rod structure in the conventional 1-3 piezocomposite causes localized deformation of the backing boundary pre-surface with the same structural periodicity. When the excitation frequency coincides with that *SAW* resonance (predominantly of a Rayleigh, or Stoneley type), the total transceiver structure and synchronized collective motion exhibit the resonance characteristics suitable for its ultrasonic operation and performance.

The effect is some similar to how the piezoelectric *SAW* devices with interdigital electrodes (*IDT*) work [16]. One of the differences is in-phase voltage applied to all piezo-rods, while in a SAW device, the counter-phase voltage additionally is applied to every other electrode strips in the electrode comb

array structure. It is also known that in the *SAW* resonator with exciting *IDT* on the piezo-substrate, the phase velocity of *SAW* is considerably reduced since the metal *IDT* implies the mass load on the substrate. As a simulation showed, there is a similar effect for the propagating *SAW*, when a loading mass shifts down the working frequency as well, from the "ideal" resonance determined by the *SAW* speed of sound.

The surface wave physically occurs in an effective pre-layer of backing, including a thin layer of hard bonding epoxy, used to connect the backing to the piezocomposite rods and filler. The condition for a relatively low-frequency *SAW* resonance (caused by its low *SOS*), in the first approximation, requires for the two adjacent rods distance to be an integer multiple to the surface acoustical wavelength:

$$A \equiv b + p \cong \lambda \cdot N \quad , \tag{3}$$

where N=1,2,3... is a positive integer; A is the pitch, as a sum of the kerf b and pillar width p. As an estimation for the SAW wave-length $\lambda=c/F$, the backing bulk SOS is roughly 1500 m/s, and the shear (SAW) SOS is estimated as $c_{saw}=\sim0.45$ $c_b\sim600$ m/s, then the backing SAW wavelength $\lambda\approx1.5$ mm for 500 kHz frequency.

2.3. FEA Simulation of SAW Resonances on Backing with 1-3 Piezocomposite Rod Structure

COMSOL (Comsol Inc., Burlington, MA) software was used to simulate the *SAW* propagation on the backing pre-surface, loaded with the periodic piezoceramic rod structure bonded on the backing boundary. In a simplified linear array of the 2D model, an elementary cell of a regular piezocomposite was considered, under continuous wave excitation (CW, applied voltage 100 V AC). The same effects are supposed to be expected as in the 3D model, but just with some shifted parameters of secondary significance. In the model design, a soft type piezoceramic PZT-5A [3], with damping coefficient Q = 20; a piezoceramic rod with basic height (length) range L from 50mc to 1.5mm, and total width from 20μ to 240μ (minimum 10μ kerf) in the fixed pitch 250μ for certainty, were used in the simulation.

The backing parameters are presented in Table II, so that for a total backing height H 4 mm, with periodicity (pitch) 250µ, it provides a longitudinal backing resonance lower 100 kHz. Both the fundamental (lowest) backing *SAW* and longitudinal resonance frequencies are relatively stable vs. piezo-rod width, as shown in Figure 2b.

For the backing SAW resonance, it has a broader peak bandwidth for wider rods.

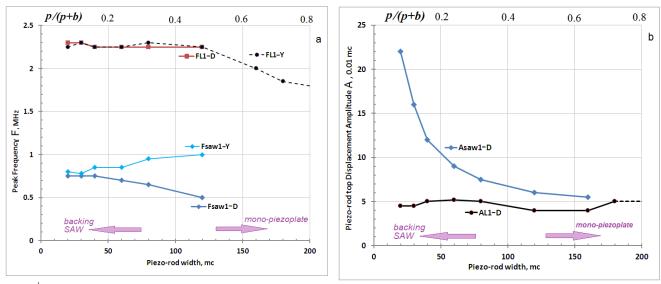


Figure 2. Simulation resonance frequencies (a) and top piezo-rod displacement (b) dependences on the rod width (pillar) under constant pitch $(p+b=250 \mu)$, as the ratio of the rod width-to-pitch (0...0.8); the piezo-rod length is 0.76 mm, voltage applied 100 V.

In Figure 2b, the data are presented for the saw1-Y and saw1-D fundamental (lowest) SAW resonance taken from the piezocomposite electrical admittance (Y = 1/Z) and from the piezo-rod top displacement (D); and for L1-Y and L1-D similarly for the fundamental (lowest) longitudinal resonance of the piezo-rod in piezocomposite; where L1 denotes the fundamental rod length vibrational mode, with three lowest SAW modes presented.

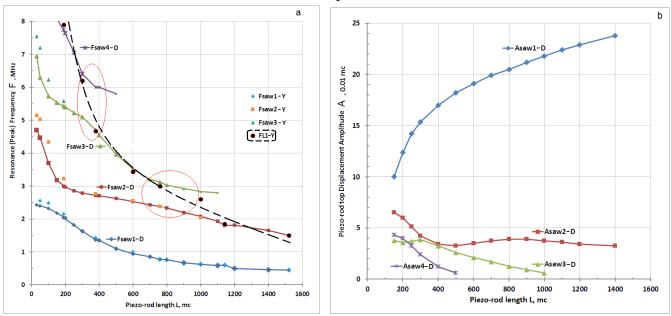


Figure 3. Simulation results for the resonance frequencies of admittance (a) and top piezo-rod displacement (b), and their dependences on the rod length, with the SAW and rod-length (piezocomposite thickness) acoustically coupled resonance branches shown inside the red circles; piezo-rod length variation for the case of constant pitch ($p+b=250\mu$) and pillar width p (25μ); voltage applied 100 V.

There are a number of SAW harmonics excited in the structure. For the baseline case of negligible piezo-rod's mass, the resonance frequencies are just directly proportional to their consecutive orders (N = 1,2,3...). For some longer piezo-rods, when their own rod's longitudinal resonance coincides with one of the SAW resonances, a coupled modes vibration takes place, as shown in Figure 3a. Under such conditions, the largest top piezo-rod displacement is provided at the lowest SAW resonance, as demonstrated in Figure 3b.

As follows from the Figure 2b data, for the piezocomposite structure with relatively narrow kerf, the top piezo-rod displacement is relatively low and close to that of a regular homogeneous piezo-plate. For a larger kerf, the fundamental *SAW* mode shows large and rapid increase in the top piezo-rod displacement at the backing *SAW* resonance - the effect directly determining the transducer higher sensitivity.

Similar data are presented in Figure 4a, but for the resonance characteristics variation under just the kerf width varying, with all other constant structural array parameters. For a relatively wider kerf, or the same larger pitch, the spectra effectively goes from a mono-frequency resonance to the multi-resonance peaks of *SAW* harmonics.

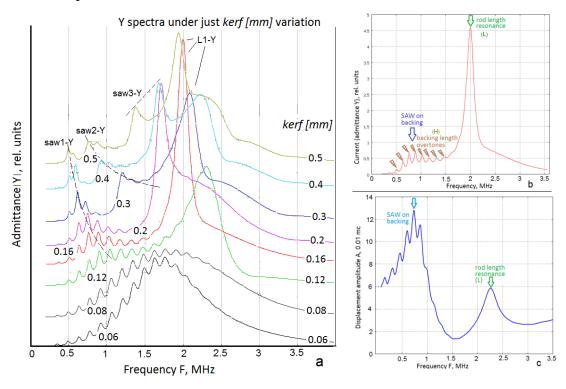


Figure 4. FEA simulated spectra (admittance IYI, CW) transformation under composite piezo-rod structural variation: a - just kerf variation (pillar width 50μ); b and c – comparative peak amplitudes admittance and top rod displacement (pitch 250μ and pillar 50μ), respectively; voltage applied 100 V; rod length 0.76 mm; structure: just piezo-rods on backing.

On the admittance piezocomposite characteristic, the SAW resonance is some hardly visible, however the piezo-rod top displacement amplitude is significantly higher than that at the

piezocomposite plate fundamental mode (Figure 4b), making it obviously beneficial at least for more efficient energy transmission into medium. Note that the fundamental *SAW* resonance on the backing boundary works like a band-pass filter for the low-frequency longitudinal backing reflection resonances, looking like a comb of resonances.

The waveform displacement and stress (von Mises) pattern at the fundamental *SAW* vibrational mode are shown in Figure 5. For a low (negligible) mass of tiny piezo-rods, the space SAW waveform is close to the ideal total sine period. The effect of piezo-rod width on the stress distribution is demonstrated in Figs. 5b-d. As seen from the simulation presented, the effective *SAW* in-depth equals to near same SAW wavelength, estimated as a motion intensity transition from the *SAW* to bulk flat waves in the backing. Note that the SAW effect does not occur for a single piezo-rod on the backing, having equal cross-areas.

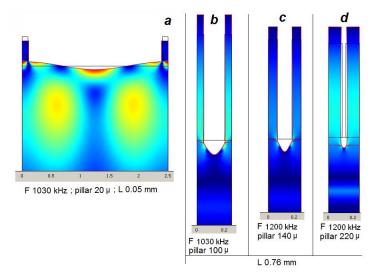


Figure 5. Space waveform displacement and stress (CW) distribution at the SAW backing resonance under composite structure (a), and their transformation under variation of piezo-rod height and width (b-c). Backing 4mm; pitch 250µ.

2.4. Basic estimations for 1-3 Piezocomposite Rod Structure with the SAW Vibrational Mode on Backing

Various types of *SAW* devices such as actuators, filters, oscillators, resonators, and sensors have been reported by researchers and are used in many industries and electronic equipment [16]. They basically consist of and are employing electro-acoustic transducers such as an interdigital transducer (*IDT*) fabricated on a piezoelectric substrate [16,17], for transmitting and receiving acoustical waves. An *IDT* consists of metallic comb-shaped electrodes normally fabricated over the surface. The mass loading effect of the *IDT* on *SAW* resonance frequency has close similarities to the SAW phenomenon on the backing under current consideration. In the literature, it was investigated the reduction of *SAW*

velocity caused by the mass loading of metal *IDT*, or as a structured mass loading along the propagation surface path.

The *IDT*s are comb shaped electrodes normally made of metal with a thickness of around 0.2 μ m over the piezoelectric substrate. *IDT*s fabricated over the substrate additionally introduce *secondary* effects such as *re-emission* (reflection) the waves. Because of high sensitivity to the mass load, the *SAW* devices have been used for developing many sensors [16,17]. Due to the mass load of *IDT* the phase velocity of the *SAW* device reduces from the *ideal* (unloaded) phase velocity. The *IDT* fabricated over the substrate affects mostly the surface wave velocity and the bulk wave velocity is not much affected since the bulk waves propagate interior to the device substrate. The velocity (c_b) of *BAW* is normally two times as higher the *SAW* velocity (c_{saw}) [3,17] in the same material (backing).

The decrease in *SAW* phase velocity caused by mass load of *IDT*-like rod is estimated below using the elementary structural unit. For the lowest first fundamental *SAW* resonance, using a correction factor based on the spring-mass model, it can be wrote

$$F_{saw} \approx \frac{c_{saw}}{A} \frac{1}{\sqrt{1 + (\sim L/A)}}$$
, (4)

where A=p+b is equal to the SAW "natural" wavelength. As an estimate, typically in practice $L\sim 2A$, then $F_L\approx \frac{c_L}{2L}$, where F_L is the rod $\lambda/2$ resonance frequency, and c_L is the longitudinal speed of sound in piezoceramic, so that typically in general $F_{saw}\approx 0.4F_L$, and lies close to the $\lambda/4$ -type ineffective piezocomposite resonance (see Table II). To the contrary, it is a SAW resonance on the backing boundary, sophisticated with an "appendant" mass of the piezo-rods.

For the rod extensional fundamental frequency higher than the lowest SAW resonance on backing $(F_L > F_{saw1})$, for simplicity considering the non-loaded condition as a reference, the rod influence on the actual SAW resonance is determined by the loading rod mass. Just note that in the opposite case, with the rod fundamental frequency lower the SAW resonance $(F_L < F_{saw1})$, the rod influence on the SAW resonance is determined by the loading rod elasticity, not its mass [18]. The resonance modes in the transitional frequency area have a dispersion character typical for modes' coupling (Figure 3). As a reference point, the "ideal" (free surface) SAW harmonic resonances, tied to the composite basic structural configuration, are determined as $F_{saw,N} \approx c_{saw}/\lambda$, with N = A = p + b as the periodic piezocomposite pitch integer multiple (harmonic order N = 1,2,3...) to the SAW wavelength.

SAW fundamental resonance frequency variation for small loading masses (3D-configuration) can be described also as in [17,18]

$$\frac{\Delta F}{F_{saw}} \approx -2F_{saw} \frac{m}{Z_b} \approx -2 \frac{c_{saw}}{\lambda} \frac{L \rho_{pe}}{\rho_b c_b} \approx -(\sim 0.7) \frac{\rho_{pe}}{\rho_b} \frac{L}{p+b} , \qquad (5)$$

where m is the rod mass per base square, Z_b and c_b are the cell acoustic impedance and bulk SOS of the backing. It particularly follows that the SAW resonance frequency vs. relative rod's width is near *constant*, as supported by the simulation results presented in Figure 2.

The latter effect partly refers to the $\operatorname{spring}(k)$ -mass(M) model, where the resonance is determined as $F_{res} \propto \sqrt{k/M}$. Then the effect of mass variation (loading), with a parameter of loading mass (M) relative to the effective mass of moving unit cell in the SAW, can be estimated as

$$\frac{\Delta F}{F_{saw}} \approx \frac{1}{\sqrt{1 + \frac{M}{\rho_b \,\lambda^3}}} - 1 \approx \frac{1}{\sqrt{1 + \frac{\rho_{pe} \, p^2 \, L}{\rho_b \, (p+b)^3}}} - 1 \approx -0.5 \, \frac{\rho_{pe}}{\rho_b} \, \frac{L}{p+b} \, \frac{p^2}{(p+b)^2} \quad . \tag{6}$$

Extending expression (5), including larger connected mass', it can be expressed as:

$$F_{saw,N} \approx N \frac{c_{saw}}{p+b} \frac{1}{\sqrt{1 + \frac{\rho_{pe}}{\rho_b} \frac{L}{(p+b)} N}}$$
 (7)

There are several particular conclusions to be noticed. The rod length derivative of the resonance frequency $(\Delta F/F)/\Delta L \sim N$, that agrees well with the simulation data of Figure 3. If the rod resonance is taken as $F_L \approx \frac{c_L}{2L}$, and backing acoustical impedance Z_b is some lower than that in piezoceramic, then the condition of maximum modes coupling with equal frequencies $F_{saw,N} = F_L$ is as follows

$$\left(2N\frac{c_{saw}}{c_L}\frac{L}{(p+b)}\right)^2 = 1 + \frac{\rho_{pe}}{\rho_b}\frac{L}{(p+b)}N.$$
 (8)

If $a = 2\frac{c_{saw}}{c_L}$; $t = \frac{\rho_{pe}}{\rho_b}$, then $\frac{L}{(p+b)}N = \frac{t+\sqrt{t^2+4a^2}}{2a^2}$, so that for high-order SAW harmonics N >> 1: $\frac{L}{p+b}\Big|_{max.\ modes} \sim 1/N$ is the condition required for the high-order modes coupling.

Based on the estimation and simulation results, with just the piezo-rods connected to backing, several performance features can be concluded related to the characteristic structural parameters:

- The *SAW* resonance frequency(s) is inversely proportional to the structural pitch. Caused particularly by the kerf increase, with the fixed rod width and length, all *SAW* resonance branches are going down, in respect to the piezo-rod thickness mode frequency in piezocomposite. The strongest *SAW* modes are when located below the thickness (rod length) resonance, and with descending *SAW* resonance intensities when above it.
- When the rod width and kerf are *varying simultaneously* and in-opposite directions, like a wider rod width under fixed pitch, then both (e.g. lowest) *SAW* and length rod resonance frequencies remain relatively constant.
- When the kerf is getting much less than the rod width (or under the kerf-to-pitch ratio reaching zero), the *SAW* resonances disappear, mostly due to the mass loading effect the total resonance spectra is

getting closer to that of the elementary monolithic rod, or plate, connected to the same cross-section backing.

- When the rod height varies, with fixed all other structural parameters, the *SAW* resonances disappear with increased rods, mostly due to the mass loading effect.

2.5. Consecutive FEA Simulation of Air-coupled 1-3 Piezocomposite Transducer with SAW Resonance on Backing

The observed new effects with *SAW* resonance on the transducer backing were COMSOL *FEA* simulated, with step-by-step added structural elements into the total rods-epoxy-backing-matching transducer structure – a structural elementary cell in the 2D model is shown in Figure 5.

The consecutive transformation of the electrical admittance spectra in the regular periodic piezocomposite assembly is shown in Figure 6. The matching-to-air boundary condition was set just through the air acoustical impedance.

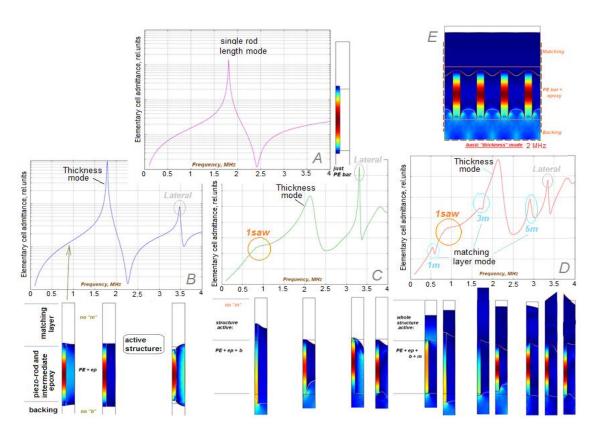


Figure 6. FEA simulation of the electrical admittance and stress, with consecutive structural transformation of the piezocomposite transducer in 2D model with periodic elementary cell containing: a single rod (*A*); just 1-3 piezocomposite plate (*B*); piezocomposite connected to the backing (*C*); full assembly with matching layer (*D*). The spectra includes the fundamental thickness mode (E), piezocomposite lateral, backing boundary SAW and matching layer resonances, at the different assembly stages. Backing 4mm; rod L 0.76mm; pitch 250µ; pillar 70µ.

Detailed description of the effects is based on the vibrational analysis of a rod connected (bonded) to the infinite boundary (backing). It determines specific features of the rod vibration, different from the case when the transverse cross-areas of the piezoelement and connected to it backing are finite and equal.

The basic stages and corresponding conclusions in the analysis of the data presented in Figure 6 are as follows. A free piezo-rod has its own fundamental length resonance (A). Then, in a piezoelement structure into a piezocomposite, additionally to that basic resonance, a higher frequency lateral resonance appears (B) [10]. And then most important, the piezocomposite plate just connected to the backing provides an extra low-frequency vibrational mode identified as a SAW resonance on the backing boundary with piezo-rods array (C). Finally, connecting the matching layer with its $\lambda/4$ -resonance provides maximum transceiver efficiency when coincides with particularly the *SAW* resonance (D). In that totally assembled piezocomposite transducer, the thickness mode per se has a conventional character of motion of the parts (E).

3. Experimental Performance of a Transceiver with piezoelectric 1-3 Composite Bonded to the Backing Layer – SAW Mode

3.1. Impedance Characteristics

For the *SAW* effect demonstration and performance comparison, two fully assembled transducers were built: one with a traditional monolithic piezo-plate, and the second with a regular periodic 1-3 piezocomposite (see Table I, sample #2), both with close overall dimensions and thickness resonances. The backing in both cases is a rod made of W powder/chips mixed with epoxy; and the matching layer is a mixture of glass-bubbles with epoxy, with its thickness corresponding to the $\lambda/4$ resonance at 500 kHz, close to the expected *SAW* resonance on the backing boundary. As shown in Figure 7, the transducer with monolithic piezo-plate demonstrates a relatively smooth resistance characteristic, without any specific resonances at low frequencies up to the fundamental thickness mode. In the case of 1-3 piezocomposite there is a characteristic peak at near 600...700 kHz identified as the backing SAW resonance, gained by the fundamental $\lambda/4$ matching layer resonance at the same frequency range, with its higher-order multiple overtones.

To preliminary evaluate the transducer acoustical sensitivity, a simple express method was used with a rubber put on the front matching layer, and then the recorded resonance peak resistance variation was used as a measure of acoustical efficiency, as demonstrated in Figure 7 with multiple

curves in the yellow area, reflecting different level of rubber damping. In the latter case, the most sensitive resonance is the $\lambda/4$ matching layer resonance located/coinciding with interpreted as a *SAW* resonance on the backing. The $\lambda/4$ matching layer resonance coupled to the backing *SAW* of the 1-3 piezocomposite plate bonded to the backing is deeply damped with quality factor near several units Q ~ 3...5, and demonstrates relatively strong sensitivity. Further experiments supported that conclusion with more details.

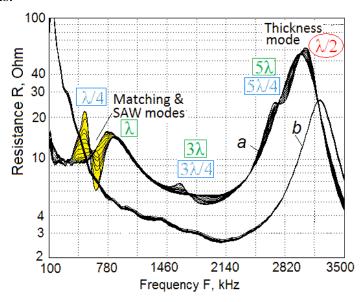


Figure 7. Comparative characteristics of the resonance vibrational modes for a regular piezoelement (b) and piezocomposite (a) with close overall dimensions, all are in the fully assembled transducers with backing and matching (at 500 kHz). The resonator OD 30 x H 0.72 mm, (a) piezocomposite sample 2 (Table 1), (b) same overall dimensions conventional piezoplate. Yellow area – most effective front surface sensitivity.

For the same sample #2 as in Figure 7, the full impedance loops of the basic thickness and SAW resonances are shown in Figure 8, demonstrating a deeply damped character of the $\lambda/4$ matching layer resonance coupled to the backing SAW. A specific of the SAW resonance is that its active resistance is close to the minimum transducer resistance (real part of the impedance), including its thickness resonance.

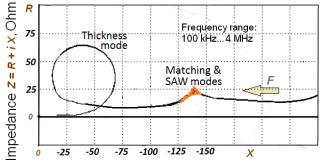


Figure 8. Resonance characteristics in the frequency range 0.1...4 MHz including the low-frequency SAW and basic thickness vibrational modes for a regular periodic piezocomposite transducer with backing and matching layer (500 kHz). The resonator OD 30 x H 0.72 mm, sample #2.

3.2. Backing SAW Resonance Mode Sensitivity in 1-3 Piezocomposite Transceiver

Three piezocomposites with different structural configurations were used in the experiments, as presented in Table I. The impedance Z characteristics just of the free original piezocomposite plates are relatively smooth below the fundamental thickness resonance, which is proportional to the plate's inverse thickness, and with some planar mode low-frequency weak "ripple", as shown in Figure 9 A. Being attached to the backing, the 1-3 regular periodic piezocomposites demonstrate complex spectra, with additional SAW fundamental mode at near 600 kHz (Figure 9 B). The data are in a good agreement with the simulation results and theoretical interpretation given to the SAW resonance, including the mass loading effect.

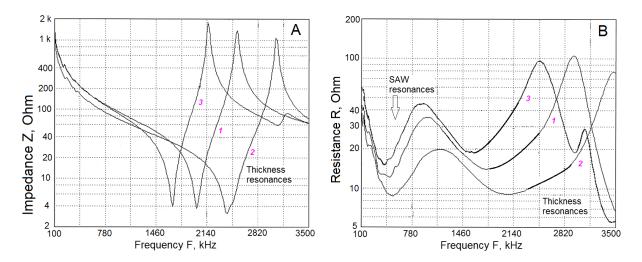


Figure 9. Effects of the backing in piezocomposites with three different thickness': experimental electrical impedance Z for initial free piezocomposite plates (A) and resistance R for those piezocomposites just attached to the 4mm backing (B); all samples as in Table 1.

The resistance R of the complex impedance Z characteristic was chosen as a parameter better reflecting the SAW resonance behavior providing deep damping. In the experiments, the backing with quality factor $Q \sim 5$ – a mixture of W powder and chips, was connected to the piezocomposite plate through a thin layer of hard epoxy – on this stage just before connecting a front matching layer.

As the acoustical strength of the SAW effect is highly sensitive to the mechanical bonding between 1-3 piezocomposite plate and backing, to demonstrate the transient resonance characteristics caused by the bonding conditions, a piezocomposite plate was put on the backing with freshly prepared glue (LOCTITE hard epoxy), with further monitoring of the resistance R characteristics transformation in the glue curing process. As shown in Figure 10, initially the piezocomposite just exhibits some damping on its basic $\lambda/2$ thickness resonance (as it's loaded with a lower-impedance epoxy liquid), then upon the bonding epoxy getting harder and solid, its intensity is further going down, while the

new SAW resonance aroused and strengthened. A novel resonance with near 2 times lower frequency is getting stronger, reaching its maximum strength when the epoxy is completely cured. This resonance is identified as a SAW vibrational type occurred on the backing boundary in the regular 1-3 piezocomposite, being out of piezoelectric rods resonance in the structure at that frequency.

Further extended experiments showed that the *SAW* resonance, and related effects, is provided only by a regular structure - the rods must be well arranged in the "dice & fill" 1-3 piezocomposite. The SAW mode on the backing boundary neither occurs with irregularly oriented and distributed rods, when piezoelectric fibers (a bunch) are imbedded randomly in a polymer matrix in the form of a 1-3 composite (for ex. [13,14]), nor with a homogeneous conventional piezoelectric monolithic plate/rod, specifically when the cross area for both piezoelement and backing are equal. The letter is demonstrated in Figure 7, where both comparative characteristics for 1-3 piezocomposite and conventional plates are presented. Note that as was found, the strongest SAW resonance is provided with a relatively hard bonding epoxy between piezoelectric structure and backing supporting a collective character of motion in the multiple regular elementary cells.

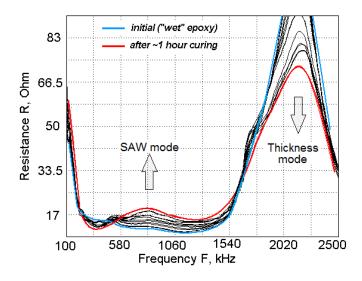


Figure 10. Comparative transient transformation of the low-frequency SAW vibrational mode resonance under curing of epoxy (increased bonding strength) between piezocomposite element and backing. The resonator OD 30 x H 0.72 mm. Rising SAW mode is identified as an additional resonance in the structure.

3.3. Pulse-Echo 1-3 Piezocomposite Transceiver Characteristics in Air at the SAW Mode on the Backing Boundary

Total transducer assembly was built with regular periodic 1-3 piezocomposites, backing and just a single front $\lambda/4$ matching layer. For generating and analyzing the acoustical waves, pulse/receiver P/R Panametrics 5077 (GE Panametrics , Lago Vista, TX) was used as a basic device, with a general

purpose oscilloscope, such as Agilent 54622 (Agilent Technologies, Santa Clara, CA) to provide pulse visualization. Impedance/Gain-Phase HP 4194A Analyzer (Hewlett-Packard, Palo Alto, CA) was used for electrical full immittance measurements, with 0.5 V max output. All measurements were performed in air under atmospheric pressure (air-coupled), at room temperature.

Signal strength measurements and Q-factor estimation were done with a transducer excited with a spike single cycle and 2% duty cycle tone burst. For special applications, where a relatively low *Q*-factor ~3...10 is required for higher resolution, the operation is provided by the *SAW* mode on the backing boundary, taking place in the experiments.

The transducer has been tested in both pulse-echo and pitch-catch modes. For a typical pulse recorded (Figure 11), with a spike input 400 V_{0-p} , the output was ~70 m V_{p-p} , with a transfer level -75 dB for a full 4" pitch-catch transmitter-receiver path in atmospheric air. The pulse has relatively short rise- and decay-times, with near 2 to 3 periods each.

In SPL measurements with a calibrated microphone, a transducer with SAW resonance at near 150 kHz (sample D, Table I) with similar input and output conditions demonstrated ~50 Pa effective acoustical pressure (pulse maximum), or SPL 128 dB re 20 μ Pa @4"@150kHz@400V_{0-p}. Estimated peak pressure sensitivity is roughly near -83dB re 1V @1 μ bar.

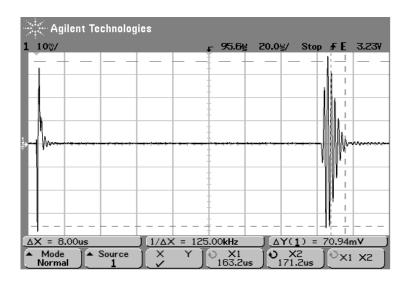


Figure 11. Typical pulse waveform of the novel piezocomposite transducer with the SAW operational vibrational mode in 1-3 piezocomposite resonator at a relatively low-frequency ~600 kHz. Sample #2 (Table I), pitch-catch with 4" distance.

The performance of the proposed novel transducer is compared with that of a conventional air-coupled transducers operating in a similar frequency range. Compared to one of the industrial prototypes such as Sonda 007CX; QMI AS400C [19] at 400 kHz, the piezocomposite transducer with

SAW vibrational mode demonstrates similar acoustical performance suitable for practical use, with a usable sensitivity and efficiency.

When the transducer was prepared in its full structure, effectiveness of its sensitivity performance was first evaluated with a simple express method - applying some soft material (like a rubber) on the front transducer surface and recording its electrical impedance variations under that influence. It's supposed that largest impedance variation is at the resonance frequency(s) of maximum transducer sensitivity (of the front surface). As shown in Figure 12, and supported by the data of Figure 7, the optimal operational frequency is near 500 kHz, with the sensitivity intensity dynamic range near ~2.4...4 times between the maximum-to-minimum peaks, with effective *Q*-factor ~3...5 [15].

The effect of matching layer thickness on the transducer performance during its operation based on the backing SAW has a typical resonance character - as was shown in an experiment, there is the optimal matching layer thickness providing maximum signal amplitude (Figure 13). It corresponds to the $\lambda/4$ matching layer thickness at the 500 kHz operation frequency of the backing SAW resonance.

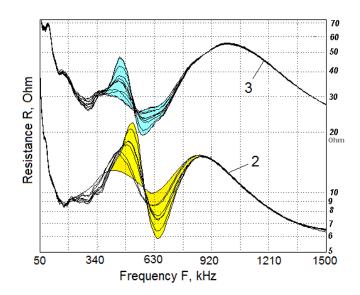


Figure 12. Effectiveness of front surface sensitivity characteristic of the low-frequency SAW vibrational mode resonance for two regular periodic 1-3 piezocomposite elements (#2 and 3, Table 1), with backing close to optimal and quarter-lambda matching layer. The area of highest sensitivity is shown in color.

The experiment was made in the pulse-echo regime, with a 2.5" SS target and the transducer using regular periodic 1-3 piezocomposite as presented in Table I, sample 2.

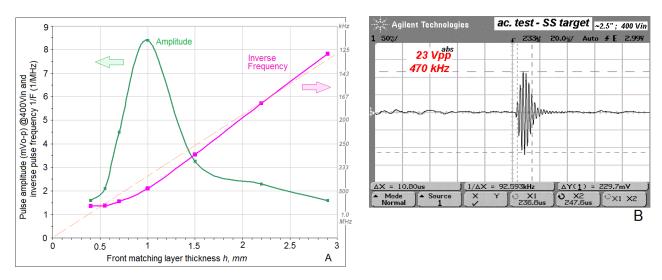


Figure 13. Experimental data of the pulse strength under variation of the matching layer thickness with SAW operational mode: A –signal amplitude and effective frequency of the pulse; B – pulse waveform with maximum output.

4. Discussion

The Rayleigh *SAW* resonance discovered in the piezocomposite backing is a powerful tool for the air (gas) transducer operation and also an interesting phenomenon for modeling. It requires first of all rigid piezorod-to-backing bonding and regular periodic structure array at least with several adjacent piezorods.

The condition for the relatively low-frequency *SAW* resonance, in the first approximation, requires for the rods periodicity to be an integer multiple to the surface acoustical wavelength on the backing boundary, which is predominantly of a Rayleigh (Stoneley) type. The surface wave physically occurs in a layer of backing, involving a thin layer of bonding epoxy between the rods and backing, with an effective depth comparable to the SAW wavelength. As was found, just a hard epoxy for bonding provides maximum acoustical signal strength. Moreover, for a piezocomposite element, it also provides better bonding quality on the boundary between epoxy filler and backing, with its extra acoustical coupling.

For the described *SAW* resonance effect, a regular periodic 1-3 (or 2-2) structure is needed. Its acoustical performance is a collective effect between at least several adjacent piezo-rods with a regular space translation. In the latter case, a wider transducer bandwidth, or even a phase array, can be achieved under special structural apodization if needed.

As was predicted and experimentally verified, no effective (i.e. collective) backing *SAW* resonance effects exist with varying planar rod dimensions, and/or irregularly distributed rods, when the piezoelectric rods (or a fibers bunch) are imbedded randomly in a polymer matrix in the form of a 1-3 composite (Figure 14). Note that the *SAW* effect does not exist also with a homogeneous

conventional piezoelectric monolithic plate/rod, specifically when the overall cross area for both the piezoelement and backing are equal.

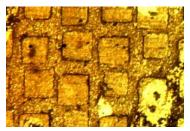


Figure 14. An example of an irregular 1-3 piezocomposite structure that causes ineffective SAW behavior: piezo-rods of different cross-sections, and their irregular (random) relative disposition.

As a possible variant, a monolithic piezoplate with the back bonding side grated can be used instead of conventional rods, or alternatively such regular grating can be put on the backing boundary surface, as shown in Figure 15. The voids can be filled with a relatively soft material. The vibrational operation is very similar, just here with larger effective mass for the "rods". It's a new joint effect with cumulative distributions of the large loading mass, as in Figure 3, and relatively small pitch, as in Figure 2.

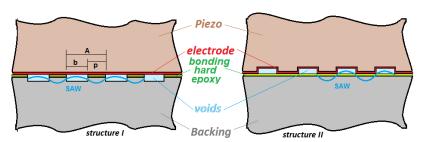


Figure 15. Alternative structures with boundary SAW effect using monolithic bulk piezoplate - with shallow regular grooves on one of the boundary surfaces in structure I and II shown.

5. Conclusions

Rayleigh surface wave (a Stoneley-type wave on the solid-solid interface) on the damping backing boundary of a piezocomposite transceiver is an effective resonating mechanism which was specifically investigated for the air (gas) transducer application. A novel vibrational *SAW* mode with a significantly lower natural resonance in a regular periodic 1-3 piezocomposite transducer was found and described. Collective *SAW* synchronized motion of the backing surface on the piezocomposite bonding boundary requires a regular (translational periodicity) rods' structure to provide a strong resonance sensitivity for the transceiver, to the contrary of the conventional thickness resonance mode approach with effective "averaged" rods behavior.

Along with less transducer weight (a thinner piezocomposite required), the described *SAW* resonance effect on the backing effectively "bridges the gap" in the transitional intermediate frequency range 100...700 kHz, problematic for the conventional transceiver designs typically with the unwantedly coupled thickness and planar resonance modes in a monolithic piezoelement with an aspect ratio close to 1. For practical evaluation, a typical piezocomposite structure (2...3 MHz thickness resonance) was used, and it provided -64 dB signal strength at the backing *SAW* 500 kHz resonance (pitch-catch, 10 cm) in air with a wide bandwidth and Q-factor near 5, being equipped with the backing and corresponding quarter-lambda conventional matching layer at that frequency.

An additional benefit of using relatively slow *SAW* waves is smaller volume and weight of the total transducer, with lower excitation voltage required because of a thinner piezoelement needed. It provides acceptable sensitivity, along with low transducer *Q*-factor and relatively wide *BW*. This is the case where strong regular periodicity greatly matters.

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Nomenclature

pillar – piezoceramic rod (with length L and width p) in piezocomposite structure, with its length-to-width aspect ratio (L/p);

kerf – cut/dicing width (b), particularly for orthogonal (square) array filled with epoxy;

pitch (kerf plus pillar width $A \equiv b + p$) – a shift between piezocomposite cuts, or structural periodicity;

CW – continues wave:

SAW – surface acoustic wave;

BAW - bulk acoustic wave;

BW – frequency bandwidth;

PE – piezoelement;

SOS – speed of sound;

SPL – sound pressure level;

SS – metal stainless steel reflecting target;

IDT – interdigital transducer;

F – current frequency;

R – electrical resistance R = ReZ component of electrical impedance Z;

saw1-Y and saw1-D – the fundamental (lowest) SAW resonance determined from the piezocomposite electrical admittance (Y = 1/Z) and from the piezo-rod top displacement (D);

N - SAW harmonic order 1,2,3...;

L1-Y and L1-D – the fundamental (lowest) longitudinal resonance of the piezo-rod in piezocomposite;

 F_{sawN-Y} and F_{sawN-D} – the SAW resonance harmonics (N = 1,2,3...) determined from the piezocomposite electrical admittance (Y = 1/Z) and from the piezo-rod top displacement (D);

 λ – wave-length, used for SAW, matching layer and piezorod longitudinal resonances;

 F_L – fundamental $\lambda/2$ rod resonance frequency;

nm – matching layer vibrational mode of the n = 1,3,5... order;

 Z_{ac} , Z_0 , Z_R , Z_m , Z_b – acoustical impedance, particularly of piezoelectric material, loading fluid, front matching and backing layers, respectively;

c, c_b , c_{saw} , c_L – speed of sound (SOS), bulk and SAW on backing, and longitudinal SOS in piezorod;

 ρ , ρ_b , ρ_{PE} – density, backing and piezoceramic density;

E, σ , Q – Young module, Poisson coefficient and material quality factor;

M, m – mass in the vibration models.