Q-factor spectrum of a piezoceramic resonator and method for piezoelectric loss factor determination

A. Mezheritsky, UFFC IEEE member
The author is with Emerson Automation Solutions, Houston, TX

Abstract – The quality factor (Q) spectrum of a piezoceramic resonator - as a Q-factor frequency dependence for the specific resonator and its vibrational modes, not for just piezoceramic material it's made of - was considered, determined under baseline low-excitation level, and analyzed with a new approach proposed. It was experimentally confirmed the theoretical prediction [1] that the resonator

Q-factor increases with frequency nearly linearly from the resonance reaching its maximum significantly closer to the antiresonance. For the iconic industrial PZT-5A piezoceramic the antiresonance-to-resonance quality factor ratio is 1.8…2.4 as much, depending on the type of vibration. As the theory states, this effect is directly related to the piezoelectric "losses" in piezoceramics, typically expressed as an imaginary part of the piezocoefficient, which has a unique property of lowering the total cumulative losses in a resonator at certain specified frequencies.

Based on the electro-mechanical Q-factor (EMQ) concept and experimental technique, a new direct, analytical and simple method was proposed, developed and used for the piezoelectric loss factor γ determination at just a single resonance frequency – it requires the resonance Q-factor and it's frequency derivative at the resonance, or the same the first and second frequency derivatives of the immittance phase at the resonance. Experimentally determined γ is close to near 0.8 of its upper (positive) phenomenological limit in the conventional PZT-5A at different vibrational modes verified.

The piezoelectric "loss" factor, getting higher and theoretically reaching closer the upper limit, can provide extremely high value of the Q-factor (nearby the antiresonance) with an order of magnitude EMQ increase. That paradox fact for the piezoelectric "losses" is a novel way of improving the piezoceramic performance and operation.

Keywords: **piezoceramics, piezoresonator, energy losses, quality Q-factor, piezoelectric loss factor, resonance and antiresonance frequencies.**

I. INTRODUCTION

Depending on the specific application, a low Q-factor piezomaterial/transducer with wider bandwidth is needed for sensors for higher resolution, and to the contrary as much as possible high Q-factor is needed for power transducers, when the heat generation due to losses restricts materials from obtaining the highest vibrational power density. Losses in piezoelectric materials phenomenologically have three components, namely, dielectric, elastic, and piezoelectric corresponding respective material coefficients with real and imaginary parts [2]. Only resonator quality factors at the resonance (Q_r) and antiresonance (Q_a) frequencies have been sufficiently studied recently. In [3,4] they were analytically derived, and later it was experimentally confirmed in [5,6]. Further, the piezoresonator EMQ concept was proposed and developed [1], with the Q-factor spectrum theoretically predicted – actually at an arbitrary frequency inside and outside the transducer working bandwidth; however, that has not been experimentally supported yet for the baseline classical low-level excitation.

Several methods of Q-factor measurement are known based on its definition and derivatives – baseline method through the decay time, as a phase frequency derivative at the phase zero-crossing, etc. A classical method, like the one with ±3(6) dB cutoff frequencies on the immittance (power) curve, is usually used to measure the frequency pairs around the resonance or antiresonance peak. The resonance and anti-resonance are both electromechanical resonances (anyway, the resonance is more "mechanical" which occurs basically under a short-circuit electrical condition), and both generate large displacement amplitudes, which can be used for actuator/transducer applications. The major difference is the driving conditions - low voltage and high current drive at low-impedance resonance versus high voltage and low current drive at high-impedance antiresonance. In most cases of PZT (lead zirconate-titanate) piezoceramics, Q_a is higher than Q_r (closer to the generalized Q_m used for industrial piezomaterial characterization), in other words the antiresonance operational mode as known has a higher efficiency [7].

A more advanced and widely used iterative automatic analysis of the resonance impedance spectra was proposed in [8] by comparing the predicted frequency dependence of impedance with an impedance spectrum measured at three points around the resonance on a sample of appropriate predetermined geometry and dimensions to derive complex elastic, dielectric, and piezoelectric properties (losses) of the resonator.

A new method was proposed for determining the electro-mechanical quality factor at frequencies other than the resonance and antiresonance points, which requires measurements of the electrical impedance, with their further computational treatment [9-11]. The mechanical quality factor is initially modeled there by the admittance phase and all three losses in piezoelectric materials. The existence of the maximum value of the mechanical quality factor at neither the resonance nor antiresonance frequencies was stated,

however the required and used supposition on the phase-frequency characteristic looks problematic for the method veracity - in the analysis, it was supposed a symmetric impedance phase characteristic inside the bandwidth, with constant slope ratios.
An “energy” method to determine the Q-factor spectrum developed mostly for high excitation levels was developed in [12,13] considering the transducer local velocity, temperature and electrical power consumption. The transducer presumably was under strong excitation level with nonlinear effects, and also the transducer end, where the displacement was measured, is not necessarily maximum displacement inside the transducer body, unless it’s the resonance. Nevertheless, it was stated [13] that the trend shows the tendency of the Q-factor to decrease as the frequency approaches a mid-point resonance inside the bandwidth. The trend could not be uncovered with present impedance technique, while the measurement at a single frequency without frequency sweeping is a benefit of the method.

In general, piezoceramics of a perovskite type have unique and intriguing properties. Just to mention a Nobel Prize in Physics (J.G.Bednorz and K.A.Muller) in 1987 for discovery of high-temperature superconductivity in perovskite ceramics (with so grainy inhomogeneous structure), that can not be fully explained theoretically so far. If superconductivity means an electrical current without losses, there is a sort of similar effect potentially allowing to get the vibration without (or extremely low) losses in conventionally lossy acoustical systems, such as piezoceramics. The effect of piezoelectric losses in piezoceramics is paradoxical as well - the piezoelectric “loss”, as one of the energy loss components providing the piezocoefficient imaginary part, reduces the total losses in a resonator – the actual dissipation at certain frequencies is significantly lower compared just to the sum of mechanical and electrical losses, under the combined electro-mechanical piezoresonator excitation with the same resonant output – and a very low loss resonant vibration is possible in traditionally lossy piezoelectrics.

The EMQ is defined [1] as quality factor $\tilde{Q} = \omega \frac{W_{kin}}{P}$ for a resonator under electrical excitation. Taking into account the internal losses $P = 0.5\,|V|^2 Re(Y)$, then for the effective kinetic energy W_{kin} vibration velocity v_{eff} (in general, velocity maximum not necessary is on the piezoelement *boundary*) can be expressed as $\omega\, m\, v_{eff}^2 = 2\,\tilde{Q}\,P$, where *m* is the resonator mass, and *V* is the voltage amplitude applied to the piezoelement. It means that the mechanical/acoustical energy efficiency (ex.: piezomotors operation, transducer transmission efficiency, etc.) for the specific piezoelement and frequency is proportional to the voltage applied squared, active transducer conductance and specifically EMQ $\tilde{Q}$ – irrespectively the resonator is electrically or mechanically/acoustically loaded. If the frequency is a resonance/antiresonance (when their impedance is pure active), the excitation source energy losses coincide with the internal piezoelement losses; if it’s not a resonance – the internal piezoelement dissipation (ultimately its temperature under operation) still is determined by $Re(Y/\omega)$, but additionally for the source of excitation, there are the reactive losses in it caused by the reactive energy circulating between the piezoelement and source.

So, the transducer mechanical/acoustical efficiency, as to the transducer internal parameters, is determined by the product of the internal losses (ultimately heat) and EMQ $\tilde{Q}$ factor – particularly, higher EMQ $\tilde{Q}$, lower transducer losses/temperature; all for the same effective vibration velocity of the transducer. And it's true at any frequency (just to emphasis - for the internal transducer performance), not necessary the resonance, or antiresonance, but inside and outside the resonance/antiresonance interval. $\tilde{Q}$ is not limited by just the piezomaterial Q_m , or even Q_r and Q_a quality factors.

The conventional equivalent circuit can not adequately explain the resonant performance of a real piezo-resonator [14]. In general, with the standard series (L_s-C_s-R_s) and parallel (C_p-r_p) branches in it, the ratio of the antiresonance-to-resonance quality factors analytically is described by the expression

$$\frac{Q_a}{Q_r} \cong \frac{F_a}{F_r} \frac{1}{1+\left(\frac{F_a}{F_r}-\frac{F_r}{F_a}\right) Q_r\, \delta} \qquad (1)$$

for $\delta^2 << 1$, which shows that practically always $Q_a < Q_r$, and even without accounting the dielectric losses the ratio Q_a / Q_r can not exceed $1+0.5k^2$, or practically 1.25 maximum. For this reason the equivalent circuit approach is not appropriate as there are numerous data with the actual ratios Q_a / Q_r reaching 2…5 times [6].

II. METHOD FOR EMQ-SPECTRUM DETERMINATION

Based on the Q-factor definition [1], two relevant methods were proposed and considered for ultimately EMQ determination. The first method is based on two frequencies of maximum (f_1)/minimum (f_2) of the imaginary immittance part (susceptance B for the resonance (F_{res}), and reactance X for the antiresonance (F_{ant})). For the generalized Q-factor of a resonating piezoelement

$$Q(F_o) \cong \frac{\bar{F}}{|f_1 - f_2|} \quad , \qquad (2)$$

where $\bar{F} = 0.5\,(f_1 + f_2)$ is close to F_{res} , or F_{ant} , frequency, and $\Delta F_Q = |f_1 - f_2|$ is the frequency bandwidth.

The second method is based on the phase frequency derivative at a zero phase crossing taking place for the admittance at resonance, and for the impedance at antiresonance. Being defined as a ratio of imaginary-to-real immittance parts, the immittance phase is to be determined in close vicinity of the resonance (or anti-resonance) as $\Delta\varphi \cong \pm 2Q\, \Delta f / F_o$, where $\Delta\varphi$ and Δf are the phase and frequency deviations out of the zero phase frequency, then

$$Q(F_o) \cong \pm 0.5\, F_o \left(\frac{\partial \varphi}{\partial f}\right)_{f=F_o} \qquad (3)$$

where F_o is the zero-phase crossing resonant frequency, generalized for F_{res} , or F_{ant} . The sign +/- depends on the phase-frequency slope at the resonance/antiresonance, which is opposite for the impedance/ admittance. Particularly, the impedance phase has a positive slope at the resonance and negative one at the antiresonance, and vice versa for the admittance. Just to notice that the phase is traditionally used on this matter, however the tangent of phase has the same properties but with some wider frequency linearity – in this sense both the parameters will be used interchangeably for the method.

Practically, both methods are not "continuous" and are based on discrete data taken from Impedance Analyzer versus frequency, but it can be controlled and made in significantly narrow frequency swiping.

Both methods are explained in details as presented in Fig. 1, for a piezoresonator made of soft PZT-5A piezoceramic, having relatively low Q-factor, chosen for the method evaluation. The impedance data, and its respective derivatives, were determined for a thick disk (type II; see also Fig. 3 and Fig. 7) resonator with a radial vibrational mode. The gray arrows and circular area show the key frequencies for the Q-factor measurements. For the used swiping settings the difference between the two methods in Q-factor determination is less than 5% .

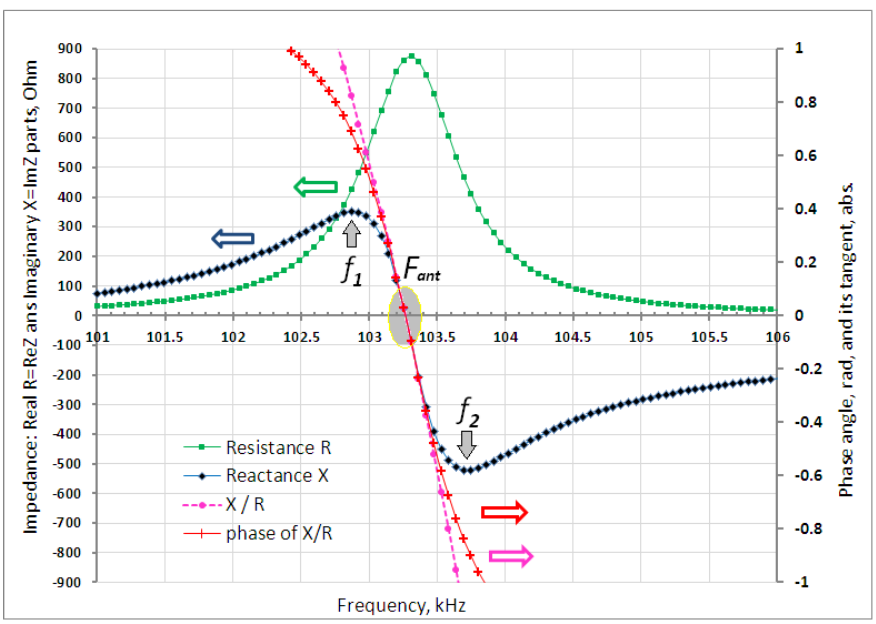


Figure 1. Basics for the Q-factor determination with typical impedance characteristics: resistance R, reactance X and the phase (with its tangent) in a vicinity of the antiresonance with in-parallel digitally connected (added) capacitance (C/C_o = 2.1); frequency discretization 55 Hz.

Determining the EMQ-factor spectrum of a piezoelectric resonator and respective measurement procedure are based partly on the following Statement.

The Statement is formulated as:

In an electro-mechanical resonator under electrical continuous-wave excitation, the resonance frequency and corresponding resonance Q-factor Q_{res} with a connected in series reactive element (capacitance, or inductance) are respectively equal to the antiresonance frequency F_{ant} = F_{res} ≡ F_o and corresponding antiresonance Q-factor Q_{ant} = Q_{res} ≡ $Q_{C/L}$ with a connected in parallel the same reactive element.

More details on the Statement application can be found in [15], and the proof is based on the following.
The resonance corresponds to a short-circuit condition of a resonating electro-mechanical system, and the antiresonance corresponds to an open-circuit condition - under both conditions ultimately there is the same resonating electro-mechanical system with in-parallel connected reactive element. The condition determines a pole for the complex resonance frequency (real and imaginary parts), with consequently equal real and imaginary parts of the corresponding complex resonance and anti-resonance frequencies - with equal frequencies itself, and with equal respective quality factors.
In case of respectively connected inductance, the equalities for the frequencies and Q-factors take place in the frequency range outside the resonance-antiresonance intervals – between lower- and higher-order adjacent harmonics, particularly between 1 and 3 harmonics.

As a reference for the piezoresonator bandwidth, in particular for the lowest fundamental mode, the antiresonance frequency shift with in-parallel connected capacitance C is $\frac{\Delta F_{ant}}{F_a} \cong -\delta_r \frac{C/C_o}{1+C/C_o}$, and the resonance frequency shift with in-series connected capacitance C is $\frac{\Delta F_{res}}{F_r} \cong +\delta_r \frac{C_o/C}{1+C_o/C}$, both satisfying the Statement.

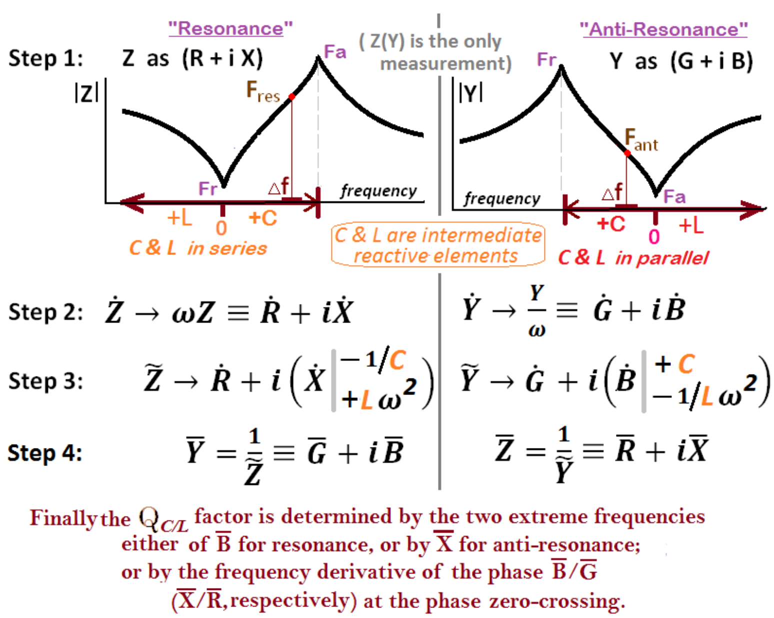


Fig. 2. Schematic for the method of EMQ Q-factor spectrum determination.
C&*L* are imitation of the reactive components connected to the piezoresonator.

An original method for the resonator EMQ spectra determination was proposed and demonstrated, particularly based on the Statement procedure. A schematic diagram for the method is presented in Fig. 2. There are two ways to measure EMQ – as a resonance or anti-resonance Q-factor, with in-series or in-parallel connected reactive element *C*, or *L*, respectively. In the method, *C* (and *L*) is just a digitally imitation intermediate parameter in the ultimate relationship between EMQ and resonant frequency, providing a shift in immittance phase.

Further detailed description of the method is provided for the first case (Fig. 2), with an in-series connected capacitance for certainty, and the second one will be further easily clear based on the diagram. As to the measurements procedure, just recording of the complex impedance (R&X), or admittance (R&B), data is needed in the frequency range under investigation, with maximum/sufficient resolution, with at least 15…25 frequency points inside the frequency $\sim F_r/Q_r$ interval of interest.

Following the schematic diagram, Step 1 is the only piezoresonator impedance measurement, the other following steps are just computer performed and driven, and ultimately can be fully executed by a respective software. Next, the frequency ω-factor multiplication is needed just to eliminate the linear frequency effect, for the "clear" resonance term not disturbed by the ω-factor in the resonator capacitive component, reducing methodological error for the EMQ determination.

The "near" resonance *EMQ* $\tilde{Q}$ factor determination covers the frequency range for the lower-resonance and inside the resonance-antiresonance interval. When the *R = ReZ* and *X = ImZ* impedance components are taken, both are multiplied by ω; further calculate $\tilde{X} = \dot{X}$ - $1/C_{ser}$, where C_{ser} is the connected in-series reactance which can be positive (for the resonance-antiresonance interval) or negative (for the lower-resonance frequencies) as an equivalent inductance. Then transform the components $\tilde{Z} = \dot{R} + i\tilde{X}$ into inverse admittance $\bar{Y}$ with the components $\bar{G}$ & $\bar{B}$, and finally find two extremes of $\bar{B}$, and calculate $Q_{C/L}$ and then EMQ $\tilde{Q}$ factors at the frequency F_{res} spectrum point according to (2) and (5).

As an alternative calculate the $\bar{Y}$ *phase* through the ratio $\bar{B}/\bar{G}$, find zero-crossing resonance F_{res} with two adjacent frequency points for the phase frequency derivative, and finally determine the same $\tilde{Q}$ factor at F_{res} by the phase method according to (3) and (5). Then repeat the steps under variation of the connected capacitance with some step, and finally calculate complete EMQ spectrum as ultimately $\tilde{Q}(f)$.

One of the largest accuracy components of the proposed methods is basically determined by the frequency discretization, taking place in a real Impedance Analyzer operation.

Lets consider the first method when the Q-factor is determined based on two peaks of the frequency extremes of the imaginary part of immittance. If that peaks frequency difference ($f_2 - f_1$ as in (2)) is divided on N intervals (absolute frequency discretization $\sim F_o$ / *(NQ)* , where Q is the current Q-factor under measurement (or the highest value for the estimations), then the average relative error in Q-factor determination is equal to N^{-1}, for example for N= 15 points the error is near 6%. That particular means that the *Q* value variation within that 6% (or within corresponding 10 absolute units at the highest determined experimental Q-factor ~180) can not be reliably detected by this method. Note that the latter can be improved with a conventional inter-points interpolation for more precise determination of the frequency extremes.

For the *Q* measurement with the phase method (3), there is an opposite effect with the upper limit absolute frequency discretization as $\sim F_o / (3Q)$ – not wider because of the phase non-linearity, and at the same time not too small because of decreased accuracy caused by random noise in the phase derivative determination. The latter can be traditionally reduced with the time averaging. The Q-factor accuracy in this case is determined as $\sqrt{2}\, d\varphi/\Delta\varphi$, where $d\varphi$ here is the phase (noise) std. deviation, or instability, and $\Delta\varphi$ is the measured phase difference for the derivative $\left|\frac{\Delta\varphi}{\Delta f}\right| \cong 2\,\frac{Q_{C/L}}{F_{res}}$ determination, so that a higher accuracy is provided for the tangent of phase, inside its much wider frequency interval with maximum linearity (Fig. 1).

To finalize the EMQ spectrum determination procedure, the following basic relationships were derived to extract the EMQ factor. As in an elementary case of LC-resonance circuit, with connected in parallel or in series *L* and *C* components, the resonant frequency is $\omega_o^2 = 1/LC$ under the condition $\omega_o L = 1/ \omega_o C$ of equal imaginary parts of impedances (admittances). At that frequency, the total circuit immittance is active, the internal energy is fully circulating between *L* and *C* reactive elements, and no energy circulating between the circuit and source of excitation. In this case the system Q-factor is determined by the ratio of accumulated energy in one of the *LC* components to the total dissipating energy, all averaged for the period. By definition based on the energy balance [1] in the electromechanical resonating piezoelectric the EMQ-factor is $\tilde{Q} = \omega\,\frac{W_{kin}}{P} = \omega\,\frac{\rho|v_{eff}|^2}{|V|^2\,\mathrm{Re}Y}$, where v_{eff} is the effective vibration velocity of the resonating body. When an ideal, with no losses, reactive (*C* or *L*) element is connected in parallel (or in-series) to the resonator, the "cumulative" resonant $Q_{C/L}$ – factor is determined as

$$Q_{C/L} = \omega\,\frac{W_{el}+W_{kin}}{P} = \omega\,\frac{|V|^2|\mathrm{Im}Y/\omega| + \rho|v_{eff}|^2}{|V|^2\,\mathrm{Re}Y} = |\tan\varphi| + \tilde{Q}\,, \qquad (4)$$

where averaged for a period W_{el} is the electric energy stored inside the "piezoresonator + reactive element" resonating system, and circulating inside the system, between the resonator and reactive element (at the resonant frequency of the system), with the losses just inside the resonator. Note that under the limit condition of unpolarized ceramic with no motion the measured Q-factor is just inverse dielectric loss factor.

The tangent of the phase angle is determined as $|\tan\varphi| = \frac{|\mathrm{Im}Y|}{\mathrm{Re}Y} = \frac{|\mathrm{Im}Z|}{\mathrm{Re}Z}$ at any particular frequency. Then finally the EMQ-factor of the piezoresonator can be determined as

$$\tilde{Q}(f = F_o) = Q_{C/L}(F_o) - |\tan\varphi(f = F_o)| \qquad (5)$$

The first term in (5) reflects the electrical energy on average stored in the "resonator + reactive element", the latter term is a module of the phase angle tangent, and its value can be also directly measured by an

Impedance Analyzer as well, and it's called there "Q-factor" however it just refers to a simple (elementary) resonating circuit (of LC type, etc.).

Further a simplifying expression $\tan\varphi \approx \varphi$ is used just for $\varphi << 1$ in a resonance vicinity close to the zero-phase. Then the system Q-factor can be determined as

$$Q_{C/L}(F_o) \cong \pm 0.5\, F_o \left(\frac{\partial\varphi}{\partial f}\right)_{f=F_o} . \qquad (6)$$

As follows from (5), under all conditions must be $\tilde{Q} > 0$ and $Q_{C/L} > |\tan\varphi|$. Also note that (6) means that $Q_{C/L}$ is not necessary a smooth function of frequency, specifically at the resonance and antiresonance with left- and right-hand different frequency derivatives, whereas the EMQ(*f*) function is smooth and differentiable.

III. Experimental data

A. Samples

For the experiments, a conventional PZT-5A (Morgan Advanced Materials, Bedford, OH) soft piezoceramic was used, most popular in the industry. To cover broader possible configurations of the piezoelement types and vibrational modes, two aspect ratios of thinner and thicker disc resonators with the radial vibration "soft, or unstiffened" k_p-mode, and a rod resonator with the longitudinal vibration "hard, or stiffened" k_{33}-mode (Fig. 3) were analyzed in the experiment – all with close fundamental resonance near 110 kHz [16]. The piezoelements have Ag-fired electrodes and were conventionally polarized in oil.

The disc resonator were fixed gently at their center to provide the radial vibrations; two thin wires were micro-soldered to the rod electrodes, and the rod was held by just the wires during the measurement to avoid extra fixing/distortion dissipation caused by unwanted vibrational modes. The wire and contact resistance in the fixture was significantly much less than the piezoresonator impedance at the resonance – just to note, otherwise the measured resonance Q-factor and especially tangent of piezoelectric losses experience significant distortion [14].

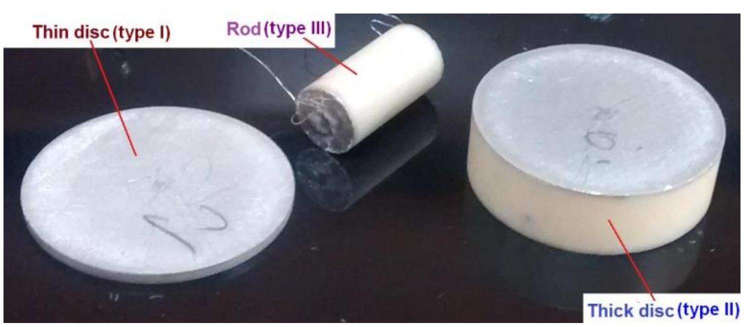


Figure 3. Piezoelements used is the experiments: thin disc (further as type I, with k_p-mode), thick disc (type II with k_p-mode), and rod (type III with k_{33}-mode).

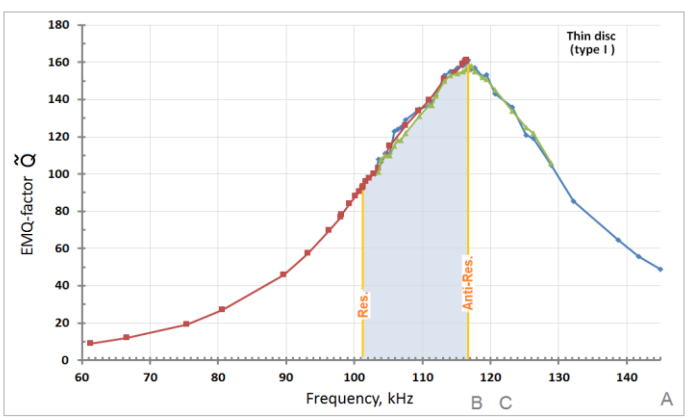
Table. Basic parameters of piezoelements

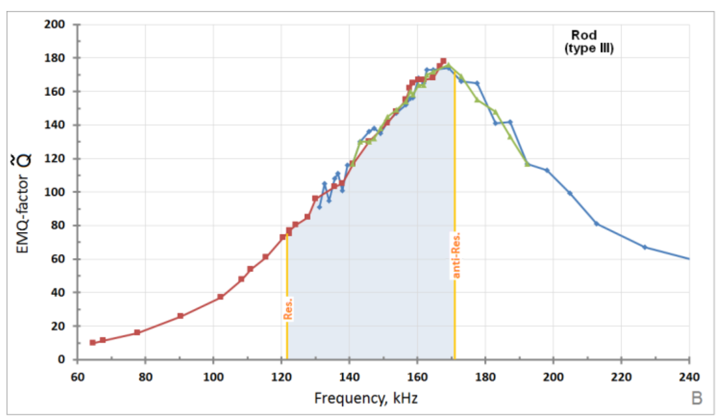
Parameter	Thin disc, Type I	Thick disc, Type II	Rod, Type III
Dimension, mm	Ø20 x H1	Ø20 x H6	Ø11 x H7
Piezocoefficient d_{33}, pC/N	390	390	440
Capacitance C_o, pF	4400	700	66
Dielectric loss factor δ (1kHz), %	1.5	1.7	1.6
Res.-antires. interval δ_r , %	15.3	18.2	39.0
Electro-mech. coupling coeff.	k_p =0.561	k_p =0.602	k_{33} =0.736
Res. quality factor Q_r $(Q_r^{-1}, \%)$	91 (1.10)	85 (1.17)	78 (1.28)
Ratio Q_a/Q_r (n=1)	1.8	1.8	2.4
Max piezoelectric loss factor γ_o , %	2.65	2.72	2.26
Piezoelectric loss factors ratio γ/γ_o	0.77	0.78	0.75

The basic standard parameters of the piezoelements are presented in Table. Note that in soft PZT5A the dielectric loss factor slightly increases with frequency near 1.35 at a hundred kHz compared to the standard 1 kHz measurement frequency. The low field excitation intensity (maximum 0.5 V applied to the resonator) was used in all the measurements presented.

B. Piezoresonator Q-factor spectrum – experimental data

Experimental data for the EMQ-factor spectrum are shown in Fig. 4 A,B,C for the fundamental radial and longitudinal modes, and in Fig. 5 A,B for their third harmonics. At the fundamental resonance, the characteristic ratio Q_a / Q_r is within 1.78…1.80 for both thin and thick discs; the ratio Q_a / Q_r for the rod resonator is higher up to 2.4, as predicted by the theory estimation [1] for “soft” and “hard” vibrational modes. Maximum Q-factor value is located inside the resonance-antiresonance frequency interval, much closer to the antiresonance frequency. That maximum exceeds adjacent antiresonance Q_a value just within near 9% at most, larger for the “hard” k_{33}-vibrational mode.





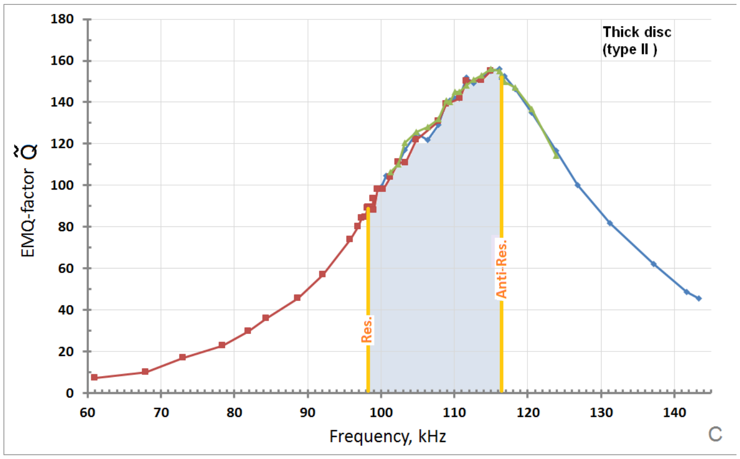
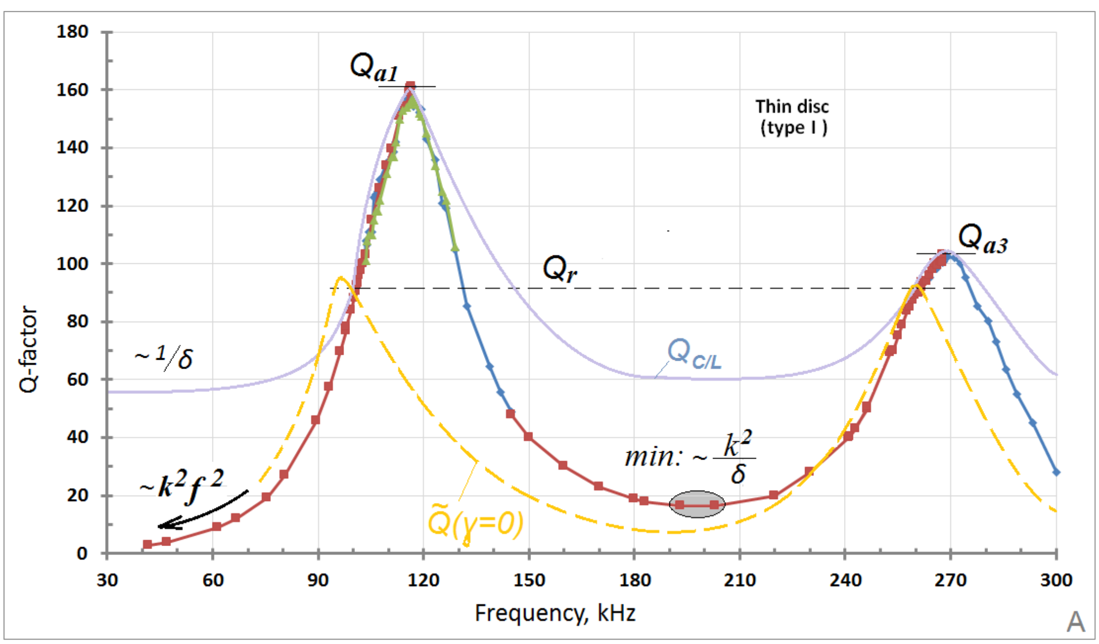


Figure 4 A-C. Experimental frequency spectra of EMQ $\tilde{Q}$ factor in vicinity of the fundamental mode: radial (soft) k_p-mode of a thin disk (A); radial (soft) k_p-mode of a thick disk (C); longitudinal (hard) k_{33}-mode of a long rod (B). See Fig. 5A for the data legend.

At the frequencies lower the fundamental resonance, for all resonator types, the EMQ decreases when frequency is getting lower, proportionally to the frequency squared, reaching just several units at tens kHz. The thin disc (type I) resonator has most "clear" radial mode resonance spectra, for this reason it was taken for high-frequency EMQ analysis (Fig. 5A). At frequencies higher than the fundamental anti-resonance, there is an EMQ minimum, with its value 17 taking place at 193…203 kHz; then EMQ is going up at the 3rd harmonic with its resonance near 262 kHz. As was estimated from the theory [1] based on determined material parameters (Table) – the minimum $\tilde{Q}$ is near 16 at high frequencies between harmonics, independently on the harmonic order. It coincides with even order harmonics which are piezoelectrically inactive.



EMQ $\tilde{Q}(f)$ method: — resonance (2), with *C&L* in-series;
— anti-resonance (2), with *C&L* in-parallel, including — phase (3) method

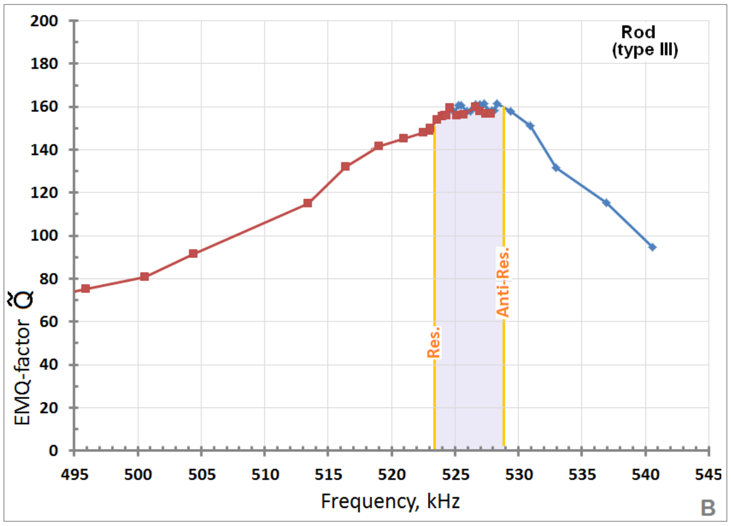


Figure 5 A,B. Extended Q-factor spectra of the thin disk type I with radial (soft) k_p-mode (A); and frequency spectrum of EMQ $\tilde{Q}$ factor in vicinity of the third harmonic of longitudinal (hard) k_{33}-mode of the long rod type III (B).
Data legend is presented in Fig. 5A.

EMQ spectra at the 3rd harmonics (Fig. 5), as was predicted [1], shows very close Q-factor values with the ratio $Q_a / Q_r \sim 1$, that basically is caused by lower effective coupling factor k^2 / n^2, where n is the harmonic number. The only difference is that for a rod k_{33}-mode resonator the third harmonic Q-factor is near the same as Q_a at the fundamental mode, and for a disc k_p-mode resonator is near the same as Q_r at the fundamental mode.

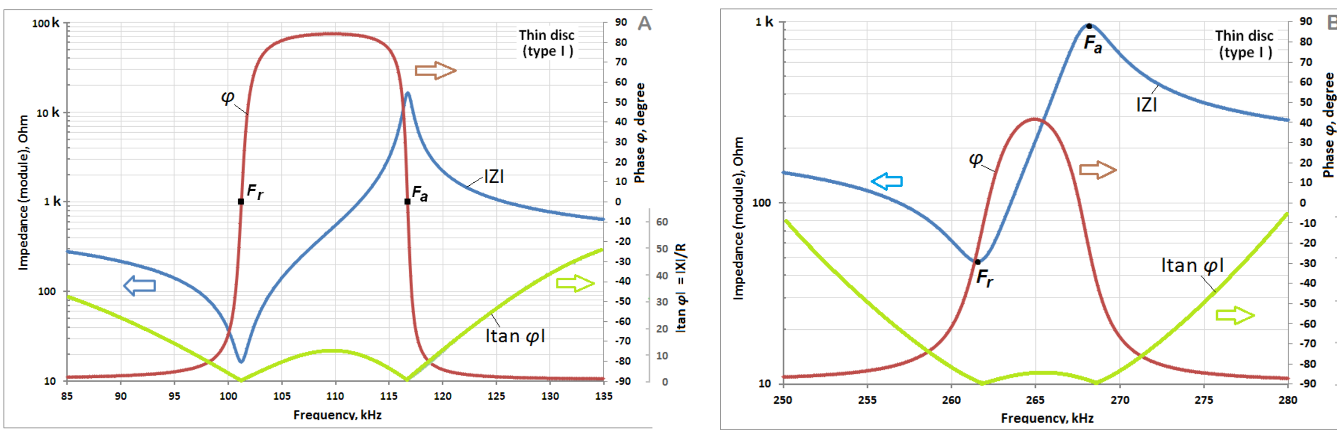


Figure 6 A,B. Impedance module IZI and phase φ, including module Itan φI, of the thin disk piezoresonator (type I) at the fundamental (A) and third (B) harmonics of k_p-type radial mode.

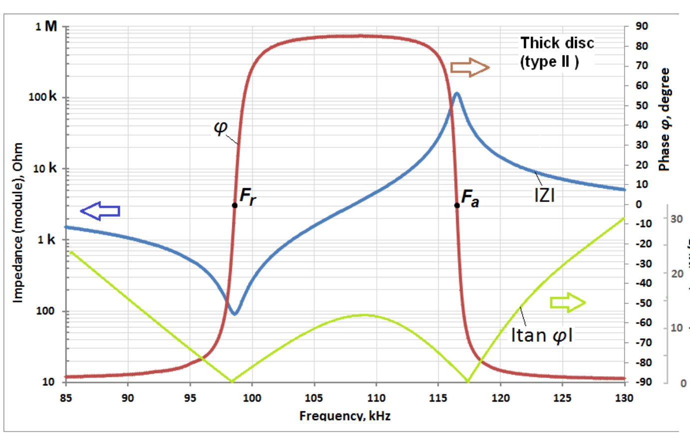


Figure 7. Impedance module IZI and phase φ, including module Itan φI, of the thick disk piezoresonator (type II) at the fundamental harmonic of k_p-type radial mode.

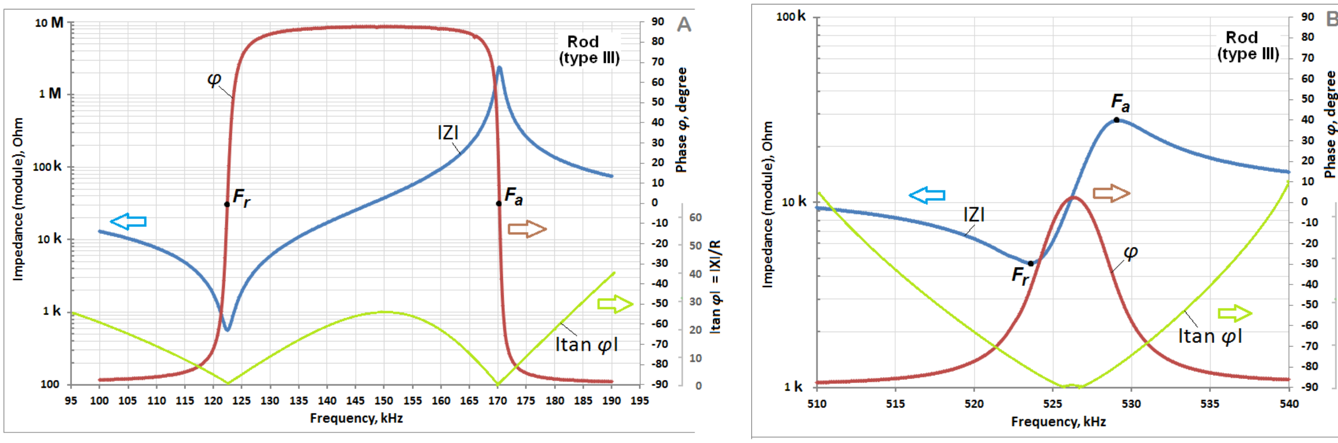


Figure 8 A,B. Impedance module IZI and phase φ, including module Itan φI, of the rod piezoresonator (type III) at the fundamental (A) and third (B) harmonics of longitudinal k_{33}-type mode.

Basic primary impedance characteristics used in the EMQ determination for all investigated resonators and vibrational modes are shown in Fig.6 – Fig.8. At the fundamental mode there is a significantly different phase slope at the resonance and antiresonance frequencies, the latter is near 2 times steeper.

IV. A METHOD OF THE PIEZOELECTRIC LOSS FACTOR DETERMINATION AT A SINGLE RESONANCE FREQUENCY

A. Piezoelectric loss factor determination through the EMQ-factor frequency derivative at the resonance.

One of the difficulties in piezoelectric loss factor determination is that it basically requires knowing all other (dissipation) parameters, that can not be determined exactly at, or close enough to the same conditions, such as for example the dielectric loss factor which can be highly dispersive and typically is determined at a quasi-static (low) frequency. The present EMQ approach and data analysis revealed the effect which specifically ties only mechanical and piezoelectric loss factors with the EMQ factor at the resonance. A new method for the piezoelectric loss factor determination can be derived based on the EMQ concept developed in [1] and is demonstrated based on the experimental data presented.

The method relates to the total losses peak asymmetry at the piezoelement resonance – the effect earlier described in [17]. As follows from [1] for the EMQ factor:

$$\frac{1}{\tilde{Q}(f)} \cong \frac{1}{Q_r} - 2\chi\,(2\,\gamma - \frac{1}{Q_r}) + 2\,\frac{\chi^2}{\delta_r}\,\delta \quad , \qquad (7)$$

where $\chi = f / F_r - 1$ is the resonance frequency displacement and $\delta_r = F_a / F_r - 1$ is the relative resonance-antiresonance frequency interval, then the piezoelectric lass factor can be determined as

$$\gamma \cong \frac{1}{2\,Q_r} - \frac{F_r}{4}\left[\frac{\partial \tilde{Q}^{-1}}{\partial f}\right]_r = \frac{1}{2\,Q_r}\left(1 + \frac{F_r}{2\,Q_r}\left[\frac{\partial \tilde{Q}}{\partial f}\right]_r\right), \qquad (8)$$

further transformed into two convenient and useful expressions

$$\gamma = \frac{1}{2\,Q_r}\left(1 + \frac{1}{2}\,\Delta F_{Q_r}\left[\frac{\partial \tilde{Q}(f)}{\partial f}\right]_r\right) = \frac{1}{2\,Q_r}\left(1 + \frac{1}{2}\left[\frac{\partial(\tilde{Q}(f)/Q_r)}{\partial(f/F_r)}\right]_r\right). \qquad (9)$$

For an easy interpretation of the method based on (9), the term with the derivative equals to the absolute EMQ variation within a half bandwidth (Q_r factor; see also Fig. 1) at the resonance. It's completely determined by the piezoelectric loss factor, together with just (mechanical) resonance Q-factor, and no dielectric loss factor is involved. Role of the Q-factor derivative in the main expression (8)-(9) significantly prevails the pure "mechanical" loss factor, and hence needs to be determined more accurately. So as follows from (9), the piezoelectric loss factor is determined just knowing the quality factor and its derivative at the resonance frequency (Fig. 4) – determined values are presented in Table.
As known [9], magnitude of the piezoelectric loss factor is restricted $|\gamma| < \gamma_o$, and for PZT-5A it can not exceed max $\gamma <$ ~0.025 depending on the vibrational mode. The determined γ values agree well with the upper limitation, but reaching it close to 0.80. Note that theoretically for the case of zero piezoelectric loss factor the EMQ-factor frequency derivative at the resonance is negative, as schematically shown in Fig. 5A (yellow dashed line).

As an accompanying effect, with a similar approach based on (7), the dielectric loss factor at the operation frequency can be determined just based on the measurements at the resonance and antiresonance - knowing the EMQ quality factor derivative at the resonance frequency, and the Q-factor "average" in the resonance-antiresonance interval (Fig. 5):

$$2\delta_r\left(2\,\gamma - \frac{1}{Q_r} - \delta\right) \cong \frac{1}{Q_r} - \frac{1}{Q_a}, \quad \text{then} \qquad (10)$$

$$\delta = \frac{1}{2\,Q_r}\left(\left[\frac{\partial(\tilde{Q}(f)/Q_r)}{\partial(f/F_r)}\right]_r - \frac{1-Q_r/Q_a}{\delta_r}\right). \qquad (11)$$

Based on the actual $\tilde{Q}(f)$ data of Fig. 4 it means that the dielectric loss factor is determined by two frequency derivatives of EMQ at the resonance vs. effective Q-factor variation inside the resonance-antiresonance interval.
As an estimation, the piezoelectric loss factor is $\gamma \cong$ 0.020…0.022 for all piezoresonators' configurations investigated, and this loss factor is the highest in magnitude among other mechanical and dielectric loss factors (Table).

Comparing the proposed method with the most advanced iterative method [8], the latter is a pure mathematical procedure, and physically it's like a "black box" – it can get any result, and there is no internal procedure to verify it, under influence of an unwanted local resonance in relatively wide frequency range required for the iteration. To the contrary, in the current method just the EMQ frequency slope, or actually immittance phase derivatives, is a measure of γ strictly at the resonance frequency.

B. *Piezoelectric loss factor determination through the second frequency derivative of the phase at the resonance.*

There is another aspect of the piezoelectric loss factor determination based on the supposition that it can be fully described by the immittance phase, including its frequency derivatives, like the Q-factor (3) determination method.

In general for a specific resonator at a certain vibrational mode, a set of the phase functions (Fig. 9) is fully described as a $\varphi(f, F_{res}(C/L))$ two-dimensional series of two frequency variables, where F_{res} is the frequency of resonance with a zero-phase (local resonance with φ = 0 for resonator impedance with in-series connected reactive element *C/L*). Based on (3) and (5), under that condition with $Q_{C/L}(F_{res}) \cong 0.5\, F_{res} \left(\frac{\partial\varphi}{\partial f}\right)_{f=F_{res}}$,

then it can be derived

$$\left[\frac{\partial \tilde{Q}(F_{res})}{\partial F_{res}}\right]_r = \pm 0.5 \left(\frac{\partial\varphi}{\partial f}\right)_{f=F_r} \pm \left(\frac{\partial\varphi}{\partial f}\right)_{f=F_r} + 0.5\, F_r \left[\frac{\partial^2\varphi(f, F_{res})}{\partial f\, \partial F_{res}}\right]_{\substack{f=F_{res} \\ F_{res}=F_r}} \qquad (12)$$

for the right- and left-hand vicinity of the piezoelement resonance with corresponding immittance phase, and finally for both Z- and Y-phase representation we have

$$\gamma \cong \frac{1}{2\,Q_r}\left(2.5 + \frac{F_r^2}{4\,Q_r}\left[\frac{\partial^2\varphi(f, F_{res})}{\partial f\, \partial F_{res}}\right]_{\substack{f=F_{res} \\ F_{res}=F_r-}}\right) \cong \qquad (13)$$

$$\cong \frac{1}{2\,Q_r}\left(0.5 + \frac{F_r^2}{4\,Q_r}\left[\frac{\partial^2\varphi(f, F_{res})}{\partial f\, \partial F_{res}}\right]_{\substack{f=F_{res} \\ F_{res}=F_r+}}\right) \qquad (14)$$

Note that the difference between the right- and left-hand second-order mixed frequency derivatives of the phase at the resonance is equal $8Q_r/F_r^2$ ($\cong 0.072\ kHz^{-2}$ for the thin disc).

In an experiment with the thin disc resonator (type I), the impedance was measured and a set of the phase branches (as the ratio *tan*φ = *X/R*) near the resonance, with in-series connected reactive element *C* (negative for connected inductance *L*), were determined as shown in Fig. 9. For the specific calculations, the imitation inductance *L* is normalized by the expression $-L = 1/(-C)\ \omega_o^2$ with negative capacitance *C*, where ω_o is the nearby resonance frequency taken just for reference – it simplifies the calculations making the resonance frequencies near symmetric to the *±C* values.

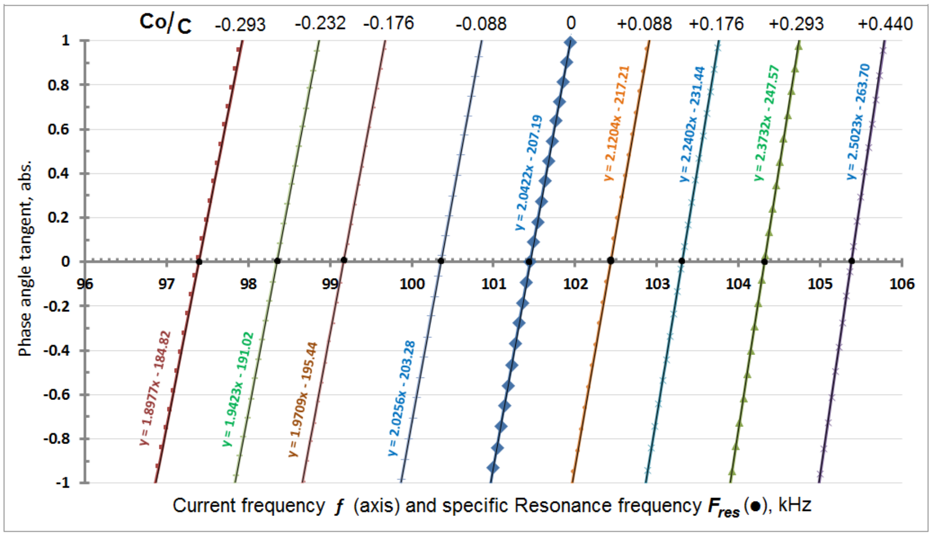


Fig. 9. Impedance phase (X-to-R ratio) with in-series connected reactive element *C*, or *L*, vs. frequency for a set of C_o/C ratios with corresponding resonances F_{res} of zero-crossings. Thin disc resonator, type I. The expressions of linear interpolation are shown in respective colors.

The first phase derivative vs. corresponding zero-crossing frequency was calculated, as shown in Fig. 10. Based on this data, the second phase derivative was determined as 0.041 kHz^{-2} and 0.111 kHz^{-2} on the right- and left-hand sides of the resonator resonance frequency, respectively. There is a jump in the derivative of $\partial\varphi/\partial f\ (F_r)$ over F_{res} frequency exactly at the resonance. With Q_r = 93 at F_r = 101.4 kHz, the calculated (13, 14) piezoelectric loss factor is γ = 0.020. Along with predetermined dielectric loss factor δ = 0.017 (11), the maximum allowed piezoelectric loss factor value was estimated γ_o = 0.025, which corresponds to the ratio $\gamma / \gamma_o \approx 0.80$.

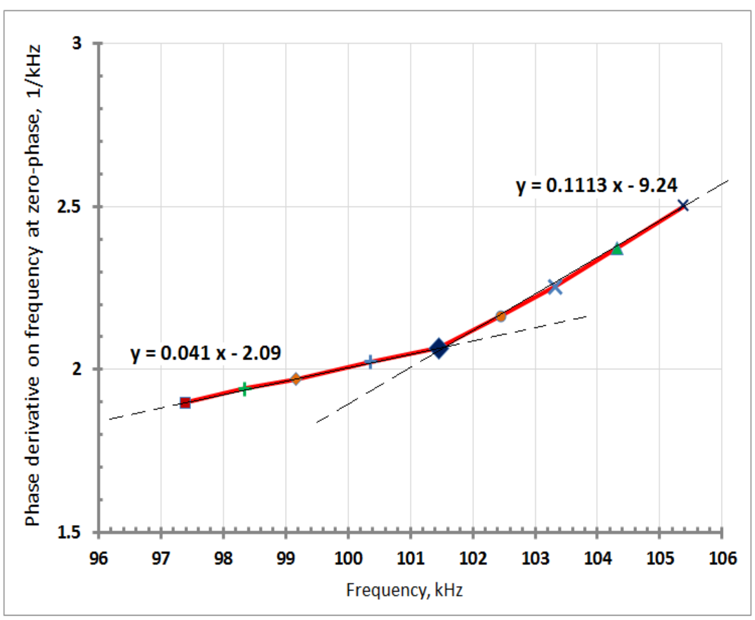


Fig. 10. The first and second (a respective slope [1/kHz^2] indicated) frequency derivatives of the phase shown in a vicinity of the resonance. Thin disc resonator, type I.

V. Discussion

The basic topic was investigated above about the EMQ spectra for different resonator types in a wide frequency range, particularly inside and near the resonance-antiresonance frequency interval. For a broader

EMQ characterization it will be discussed for a loaded resonator and shown how the EMQ reaches frequencies well below the lowest fundamental resonance, and what the minimum EMQ value(s) between two adjacent harmonics is. Another mostly theoretical aspect is discussed here what happens to EMQ when specifically the piezoceramic polarization is reaching zero.

If a piezoresonator is electrically or mechanically (acoustically) loaded, the resonator internal loss (ultimately heat) is anyway determined by the resonator EMQ factor, not the Q-factor of the whole loaded resonator accounting also the electrical/acoustical energy output. As was experimentally confirmed the maximum EMQ takes place very near the antiresonance, but a practical recommendation can be made to use the antiresonance with no reactive current component to minimize the total losses, including specifically the losses inside the electrical power source.

According to [1], the EMQ exactly at the (mechanical) *resonance* is expressed as $\tilde{Q} \propto \frac{k^2 Q_r^2}{\delta + k^2 Q_r}$, which is equal to $Q = Q_r$ for $k^2 \gg \delta / Q_m$, or approximately $k \gg 1/Q_m$. Just for lower piezoactivity, EMQ $\tilde{Q} \to 0$ for $k \to 0$ under non-polarized state when there is no kinetic energy under electrical excitation, and the dissipation is caused just by the non-zero dielectric loss – if it's zero, then always $\tilde{Q} \to Q_r$ at the resonance (all under low electrical excitation field). Note that according to the phenomenological restrictions [9], there are no piezoelectric losses when the dielectric losses are zero.

Following [1], the EMQ far below the fundamental resonance F_{r1} , where the conductance G is reaching $\omega C_o \delta$ as in a regular capacitor, EMQ is decreasing reaching zero $\tilde{Q}(f) \to 0$ under $f \to 0$ approximately as $\tilde{Q}(f \to 0) \propto \sim \frac{f^2}{F_{r1}^2} \frac{k^2}{\delta}$ (Fig. 5A).

Based on [1], minimal EMQ $\tilde{Q}$ value *between harmonics* (close to the even-order harmonic where the resonator conductance G ≈ $\omega C_o \delta$) is constant on the harmonics order and equals $\tilde{Q}_{min} \cong \frac{0.5\, k^2}{\delta - k^2 (2\gamma - 3/2 Q_r)} \sim \frac{k^2}{\delta}$ (Fig. 5A) , which is in good agreement with the experiment. Its value can reflect the δ frequency dependence taking place in practice. To the contrary of high-Q piezotransducers, the extremely low-Q resonating system as described above can be used widely in sensing systems, for example.

VI. CONCLUSIONS

In this study, the electro-mechanical Q-factor (EMQ) has been investigated on the basis of the resonator immittance analysis, and the EMQ factor spectra and its maximum/minimum values were specified in a wide frequency range, from quasi-static up to high-order harmonics. By a new proposed method to determine the resonator EMQ quality factor spectrum, experimentally confirmed the theoretical prediction [1] stating there is the highest quality factor located very near the antiresonance frequencies within the resonance bandwidth, which is almost double the value at the fundamental resonance in PZT-5A type piezoceramic.

The theoretical [1] and practical evidence obtained in this paper will benefit the selection of the optimal operation frequency for piezoelectric devices from the loss reduction/addition aspects, based on the existence of the special driving frequency where EMQ factor reaches maximum/minimum values, respectively. More importantly, the explanation provided in this study will help researchers initially understand the reason for identifying the most efficient working frequency. General mechanism based on existence of the piezoelectric loss component with its specific properties was described for the resonator EMQ spectra at different vibrational modes, which can be practically useful in piezoelectric designs with both soft and hard piezoceramics providing high and low energy dissipation and damping. No significant difference in the EMQ-spectra was found between the rod piezoresonator with the “hard” longitudinal vibration mode, and thin and thick disks with the “soft” radial vibration modes.

As an additional benefit, based on the fact of linear EMQ dependence near the resonance, a new method for piezoelectric loss factor determination was proposed and developed which requires just frequency derivatives determination of the immittance phase at the resonance frequency. Based on the resonator EMQ spectrum data and its full characterization at the resonance, the piezoelectric loss factor was determined as close to 0.77±0.02 of the upper phenomenological limit, that is playing a critical role in the EMQ spectra character and determining optimal practical solutions. Note there is no need in a predetermined known (usually) admittance function of an ideal vibrational mode as in the iterative method [8].

REFERENCES

1. A.Mezheritsky, “Elastic, dielectric and piezoelectric losses in piezoceramics: how it works all together,” Ultrason. Ferroel. and Freq. Control, vol. 51, no. 6, pp. 695 - 707, 2004.
2. R. Holland and E. P. EerNisse, *Design of Resonant* Piezoelectric *Devices*. MIT Press, Cambridge, 1969.
3. A.Mezheritsky, “Quality factor of piezoceramics,” Ferroelectrics, vol. 266, no. 1, pp. 277-304, 2002.
4. A.Mezheritsky, “Quality factor of piezoceramic resonator.” Radio Engineering / Radiotehnika (in Russian), no. 11, pp. 25 – 27, 1987.
5. A.Mezheritsky, “Quality factor of piezoceramic resonator operating at overtones.” Radio Engineering / Radiotehnika (in Russian), no. 4, pp. 74 – 77, 1990.
6. G.Liu, S.Zhang, W.Jiang, W.Cao. “Losses in ferroelectric materials,” Materials Science and Engineering R, vol. 89, pp. 1-49, 2015.
7. A.Mezheritsky, “Efficiency of excitation of piezoceramic transducers at antiresonance frequency,” Ultrason. Ferroel. and Freq. Control, vol. 49, no. 4, pp. 484 - 494, 2002.
8. J.G.Smits. *Eigenstates of Coupling Factor and Loss Factor of Piezoelectric Ceramics*. Technische Hogeschool Twente, 1978, 205 p.

9. W.Shi, H.Zhao, J.Ma, Y.Yao, K.Uchino, "Investigating the frequency spectrum of mechanical quality factor for piezoelectric materials based on phenomenological model," Jap.J.Appl.Physics, vol. 54, no. 10, article no. 101501, pp. 1 - 5, 2015.
10. W.Shi, H.Shekhani, H.Zhao, J.Ma, Y.Yao, K.Uchino, "Losses in piezoelectrics derived from a new equivalent circuit," J.Electroceramics, vol. 35, pp. 1 - 10, 2015.
11. W.Shi, H.Zhao, J.Ma, Y.Yao, "Optimal working frequency of ultrasonic motors," Ultrasonics, vol. 70, pp. 38 - 44, 2016.
12. H.Shekhani and K.Uchino, "Evaluation of the mechanical quality factor under high power conditions in piezoelectric ceramics from electrical power," J.Eur.Ceram.Soc., vol. 70, pp. 1 - 4, 2014.
13. H.Shekhani and K.Uchino, "Characterization of mechanical loss in piezoelectric materials using temperature and vibration measurements," J.Am.Ceram.Soc., vol. 97, no. 9, pp. 2810 - 2814, 2014.
14. P.Kandiba and A.Mezheritsky, " Energy losses in piezoceramics and their accounting in the equivalent circuit of resonator." Radio Engineering / Radiotehnika (in Russian), no. 10, pp. 92 – 94, 1983.
15. Patent SU 1823763 A1. Method of making of piezoceramic element for frequency generator. A.Mezheritsky, 1992.
16. IEC Publication 483. *Guide to Dynamic Measurements of Piezoelectric Ceramics with High Electromechanical Coupling*, 1978.
17. A.Mezheritsky, "Energy losses in piezoelectric ceramics under electrical excitation," Electricity (in Russian), no. 10, pp. 65 – 67, 1984.

<u>Nomenclature</u>:

F_{res} (F_{ant}) – "current" resonance(antiresonance) frequency (generalized one designated as F_o), with a connected in-series (in-parallel) *C* or *L* reactive elements as in the method;

$Q_{C/L}$ – corresponding resonance(antiresonance) Q-factor (generalized Q-factor *Q*), with a connected in-series (in-parallel) *C* or *L* reactive element;

$\tilde{Q}$ – electro-mechanical (EMQ) Q-factor;

F_r , Q_r – "intrinsic" resonance frequency and Q-factor of piezoelement at the resonance;

F_a , Q_a – "intrinsic" antiresonance frequency and Q-factor of piezoelement at the antiresonance;

C_o , $C(L)$ – capacitance of the piezoelement and connected to it in-series $C = C_{ser}$ (or in-parallel $C = C_{par}$) capacitance (or inductance *L*);

δ_r – relative resonance-antiresonance frequency interval for a specific piezoelement vibrational mode;

k – generalized electro-mechanical coupling factor, depending on the vibrational mode;

k_p and k_{33} – specific planar and longitudinal coupling factors;

Q_m – "mechanical" quality factor of piezoceramic material specified by manufacturer;

δ – dielectric loss factor;

γ , γ_o – piezoelectric loss factor and its maximum phenomenological limit ($\sqrt{\delta/k^2Q_m}$) , respectively;

$Y = G + iB$ – complex admittance, with real conductance *G* and imaginary susceptance *B* components;

$Z = R + iX$ – complex impedance, with real resistance *R* and imaginary reactance *X* components ($i = \sqrt{-1}$);

For all them together, $Y = 1/Z$ and both are called also immittance.

φ – immittance phase angle;

I$\tan\varphi$I – immitance Q-factor $|X|/R = |B|/G$, measured by Impedance Analyzer;

$\omega = 2\pi f$, *n* – circular frequency and harmonic order number;

f_1 , f_2 – extreme frequencies of susceptance *B* at resonance , or of reactance *X* at antiresonance;

χ – frequency deviation relative to the resonance (antiresonance);

L_S, C_S, R_S and C_p, r_p – inductance, capacitance and resistant elementary components (series and parallel branches) in the conventional equivalent circuit of a resonator;

V – voltage applied to the resonator.

November 2016